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Faculty of Economics, Commercial Sciences and Management

Department of Economic Sciences

Microeconomics 1

Lessons and applications

Handout prepared by: Dr. Boudiaf Hafid

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Contents

Introduction

The primary purpose of this publication is to present a study on the subject of microeconomics in a simplified form, in accordance with the microeconomics scale program for first-year students in economic sciences, commercial sciences, and management sciences. It aims to introduce the student to the basic principles of unitary analysis, which requires the student's understanding of various The theories and content of the concepts and definitions used in this field on the one hand, and his familiarity with some relevant mathematical and statistical concepts, such as functions and their related continuity, derivation, cardinal functions, etc., as well as how to deal with various data in terms of graphical representation and statistical analysis of them.

This document included eight chapters that included the essence of economic theory, starting with an introductory chapter that dealt with some basic concepts, especially with regard to defining the economic problem in terms of its origin, nature, and method of treatment. We worked to clarify mathematical relationships and theoretical graphical forms with applied examples that allow the student to learn how to deal with the problems that posed by this subject.

1 .Economics and the economic problem

Economics is concerned with studying human behavior towards meeting unlimited human needs, using available scarce economic resources. It is clear from this concept that the nature of the economic problem that all societies face, to varying degrees, is the presence of limited quantities of economic resources, matched by unlimited needs for... Goods and services that members of society want to obtain, and this is what economists express as the problem of scarcity. Therefore, the available economic resources must be optimally exploited in order to produce the largest possible quantity of goods and services according to a specific system of priorities.

1.1 Definition of economics

Economics is known as one of the social sciences that is concerned with studying how society uses its relatively limited economic resources to produce and satisfy multiple human needs.

It gives basic economic analysis tools, which enable understanding and interpretation of various activities related to consumption, production and saving. Economic analysis is divided into two basic parts: macroeconomics and microeconomics.

2.1 Macroeconomics and Microeconomics

Economics is studied at two basic levels: at the level of the decision-making unit, that is, at the level of the consumer or business establishment, and how these units interact in the market for each good or service, determining the quantity required and produced, and determining its price, and how they interact in the markets for the factors of production, determining the prices and quantities of each. This branch is called microeconomics. It also studies economics at the macro level, where it is concerned with aggregate demand, output or aggregate supply, the general level of prices, inflation, employment level, and economic growth. This branch is called macroeconomics.

3.1 Human needs and their characteristics

From the previous definition of economics, it is clear that it focuses on how to meet and satisfy human needs for goods and services, and because these goods and services are relatively rare due to the scarcity of economic resources, they are therefore also called economic goods and services. Human needs have characteristics that we summarize as follows:

1.3.1 Diversity: Human needs in any society are characterized by multiplicity and abundance. They are represented by vital and basic needs, such as the need for food, housing, and clothing. Then comes the need for treatment and education until we reach the need for entertainment and the satisfaction of leisure time. Some of them are tangible, such as goods, and some are intangible, such as services.

Also, some of these needs are necessary and some are luxury, and this classification depends on the living conditions of individuals. What is necessary for one person may be a luxury for another person. Tourism, for example, may be necessary for those with high incomes, while it is a luxury for those with middle incomes, and there is no need for it for low-income peoples, this applies to the same person before and after his living conditions improve, and what is necessary in one country may be a luxury in another country.

2.3.1 Diversity: Just as human needs are multiple, they also vary. The difference in tastes and the diversity of preferences necessitates the economy to diversify into products and their forms of goods and services, and needs are generated from each other. For example, the need for tourism generates the need for means of transportation, and means of transportation in turn vary from bicycle, car, train, plane, etc., and in their diversity they provide multiple options according to different tastes.

3.3.1 Renewal: Needs are also characterized by being renewable. Most of the goods and services that a person needs are renewed continuously and periodically. Cars of all types and shapes, mobile phones, and computers are the best examples of this. What was acceptable in the past may not be so at the present time because renewal and continuous change.

However, since human needs are distinguished by these characteristics, can the economy provide all these goods and services with the economic resources that society possesses that are characterized by relative scarcity?

Before answering this question and what emanates from it, we must first learn about economic resources.

4.1 Economic resources

Economic resources are called factors of production and sometimes inputs. Generally, they mean the factors used in the production process. They are divided into four elements: labor, capital, land, and the entrepreneur. It is limited and insufficient to satisfy all human needs, as it is characterized by its relative limitation and scarcity, and the criterion of scarcity is the presence of a price in exchange for obtaining it, in contrast to free resources that are available in nature in abundance and are sufficient to satisfy all human needs, and no price is paid in exchange for obtaining them, such as sunlight and air. .

1.4.1 Work: It means every mental or muscular effort exerted in the production process. Therefore, it is often called human resources. As a result of its participation in the production process, this resource obtains a return called wages.

2.4.1 Capital: It means all the elements that a person has produced to assist him in subsequent production processes, such as machinery, equipment, roads, bridges, means of transportation, buildings...etc. These items are called capital goods or investment goods. Capital receives a return in return for its participation in the production process, called interest.

3.4.1 Land : The concept of land in economics includes what is on it and what is inside it, such as arable lands, rivers, oceans, seas, minerals, fuels, forests, etc., which can be used in the production process. The land element has a return as a result of participation in the production process, called rent.

4.4.1 The organizer: The organizer is considered one of the most important elements of production because he is the mastermind who assembles the elements of production and undertakes the process of mixing them to produce goods and services. He also draws up strategic policies for the facility, such as determining the type, quantity, and price of the good or service he wishes to produce, and obtains a return for that called profit.

5.1 Economic problem

In societies' pursuit of achieving well-being for their individuals, regardless of the prevailing economic thought in them, they face a problem represented by the multiplicity, diversity, and renewal of needs for various goods and services, and at the same time they have relatively limited economic resources that do not meet all of these needs, and therefore society has had to choose the quality and the quantity of goods and services that it can produce, with the economic resources it has available.

The process of selection is an urgent necessity that requires any society to sacrifice goods and services of lesser importance in exchange for obtaining goods and services of greater importance, which it can obtain with the resources it has available to produce them. For this reason, economists see the necessity of confronting three important and fundamental economic questions in the search for appropriate solutions. The economic problem is: What do we produce? How do we produce? For whom do we produce?

1.5.1 What do we produce?

To answer this question, society must choose from a very large group of goods and services that it deems necessary to produce in quality and quantity in accordance with its economic resources.

2.5.1 How do we produce?

The answer to this question involves searching for the optimal method or the most efficient method among the multiple production methods, which ensures the exploitation of available economic resources to produce the largest possible quantity of goods and services to meet the greatest number of needs.

3.5.1 Who do we produce for?

That is, research into the segments of society and even the communities targeted by this production and how to distribute it.

2 .Demand theory

1.2 Definition of demand: Demand expresses the different quantities that consumers are willing and able to purchase of a commodity, at different prices and during a certain period of time, with the rest of the determinants of demand remaining constant.

2.2 The Law of Demand: This law states that there is an inverse relationship between the quantity required of a certain commodity and its price, with the rest of the determinants of demand remaining constant. That is, the higher the price of the commodity, the lower the quantity demanded of it, this is expressed as a contraction in demand, and as its price decreases, the quantity increases, and this is expressed as an expansion in demand.

The inverse relationship between the quantity and price of a good can be expressed either as a demand schedule, demand curve, or demand function.

3.2 Demand schedule: It is a table that shows the various quantities demanded by the consumer at different possible prices of the good for one consumer (Table No. 2-1).

Table No. (2-1): Individual application table on the meat commodity within a year

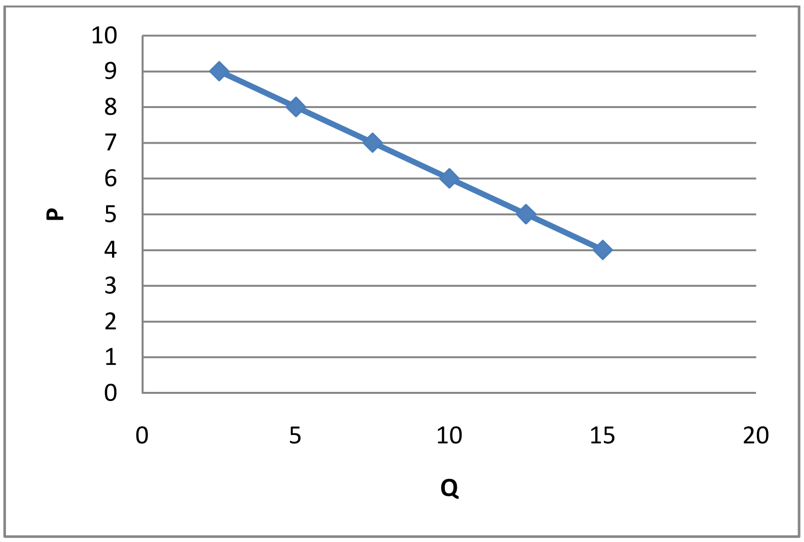
Quantity required Q (kg)	Price P (dinar)
15	4
12.5	5
10	6
7.5	7
5	8
2.5	9

We notice from the table that the law of demand is embodied, where an increase in the price, for example, from 4 to 9 dinars leads to a decrease in the quantity demanded from 15 to 2.5 kg.

Note: A market demand table is a table that includes the sum of all individual orders at their corresponding prices in the market for a given commodity.

4.2 Demand curve: It is a curve that shows the inverse relationship between the various quantities required of a particular commodity, which are represented on the interval axis, and the corresponding potential prices, which are represented on the ranking axis (Figure 2-1).

Figure (2-1): Individual demand curve for meat during a year



This curve represents the data of the demand schedule for the meat commodity, and the demand curve is characterized by a slope from the top left to the bottom right, that is, it has a negative slope, because there is an inverse relationship between the quantity demanded and the price, and it is not required to be linear as in this case, it can be non-linear.

5.2 Demand function : It is a function that links the quantity consumed of a particular commodity as a dependent variable, with the most important factors affecting it as independent variables, the latter of which are called determinants of demand.

$$Q_d = f(P_A, P_B, P_C, \dots, R)$$

Where: : P_A the price of the commodity studied, P_B: the prices of substitute goods, P_C: the prices of complementary goods, R: income.

To determine the effect of these factors on the required quantity, each factor is isolated by studying its changes in the required quantity while assuming all other factors to be constant, then studying another factor... and so on, then generalizing, and this is called abstracting scientific theory.

6.2 Exceptions to the Law of Demand : The Law of Demand may not be achieved in some cases in real life, as a rise in price leads to an increase in the quantity demanded, and a decrease in it leads to a decrease in the quantity required, and these cases include:

1) Expectation of a decrease or increase in the supply of the commodity: If consumers expect a decrease in the supply of the commodity, they will increase their demand for it for fear of losing it from the market, which leads to an increase in the price accompanied by an expansion in the quantities required. Conversely, if they expect an increase in the supply of the good, they will reduce their demand, which will push the price down and be accompanied by a contraction in demand.

2) Expectation of a decrease or increase in the price of a good: The price of a good may fall, and this decrease leads to a decrease in the quantity demanded if consumers expect the price to continue to fall. Conversely, if they expect the price to continue to rise, they will increase their demand for the commodity.

3) Belief in the quality of the commodity: A group of society may increase its demand for a commodity due to its high price, because it believes that the high price is evidence of the quality of the commodity.

4) Giffen's Puzzle: The rise in the price of a basic commodity, such as bread, for example, leads to a decrease in the purchasing power of poor families, which leads them to reduce the consumption of other foodstuffs, such as meat, and increase the required quantity of bread. If its price decreases, purchasing power increases, so families reduce bread consumption in order to increase consumption of better goods.

7.2 Demand determinants

1) Commodity price: The law of demand does not express a mere mathematical relationship between quantities required and prices, but rather expresses a behavioral relationship based on a logical interpretation of the rational behavior of the consumer, which involves satisfying the maximum possible needs within the limits of income and commodity prices, but the consumer’s limited income is faced with His unlimited needs here express scarcity, which forces the consumer to make choice decisions in order to maximize his utility. The reason for the inverse relationship between quantities demanded and the price of the commodity is due to both the substitution effect and the income effect.

a. Substitution effect: When the price of a certain commodity decreases, and the prices of other commodities remain constant, the demand for these commodities increases due to the decrease in their price, and the demand for other commodities decreases, thus they have replaced these commodities, and vice versa, if their price increases while the prices of other commodities remain constant, it will decrease Demand for it due to its high price and its replacement by other commodities.

B. Income effect: If the price of a commodity decreases, the consumer becomes able to purchase larger quantities of it and other commodities, because his purchasing power increased with the decrease in price, meaning his real income increased despite the stability of his nominal income. His purchasing power also decreases if the price of the good increases, forcing him to order smaller quantities.

2) Income: Income is the sum of the amounts allocated by the consumer for spending. It is not limited to work wages, but rather includes income from all sources, including grants, gifts, government and private subsidies. The relationship is direct between income and the required quantity of ordinary goods. The higher the income, the higher the demand for the good, and the lower the income, the lower the demand for it, assuming the remaining factors affecting demand remain constant.

Note: Goods were divided according to the change in demand for them as a result of the change in income into:

Natural goods: These are goods for which the demand is directly proportional to income, that is, income and demand for the good change in the same direction, by increase or decrease.

Inferior goods: These are goods for which the demand is inversely proportional to income. If income exceeds a certain limit, the demand for these goods decreases, or turns into better goods.

3) Prices of other goods

Commodities are divided according to how demand is affected by price movement into:

a. Substitute goods: are goods that one of them can replace another to satisfy the same need. If the price of one rises, the demand for the other rises, such as coffee and tea. Because the consumer is looking for the highest level of satisfaction, he therefore tends to demand alternative goods whose price has become relatively cheaper. The consumption of the commodity whose price has increased is reduced, but the amount of substitution is due to the degree of substitution between them.

B. Integrated goods: These are goods in which one cannot be consumed without the other, so the demand for them increases or decreases, such as tea and sugar, or a car and gasoline. An increase in the price of one of them leads to a decrease in demand for it, and this decrease in demand for it leads to a decrease in demand for its complementary good.

C. Independent goods: These are goods for which there is no relationship between the rise in the price of one of them and the demand for the other good, such as cars and tea.

4) Consumer tastes: Consumer tastes and preferences change over time as a result of changing consumer habits in society. If tastes are in favor of the good, demand for it increases at the same price, and vice versa, as demand for it decreases if tastes are not in favor of it.

5) Consumer expectations: Expecting the loss of a commodity from the market or increasing its price in the future leads to an increase in demand for it at the present time, and vice versa, anticipating an abundance of the commodity or a decrease in its price in the future leads to a decrease in demand for it at the present time.

6) Number of consumers: Since the market demand for a commodity is the sum of individual demands, an increase in the number of consumers inevitably leads to an increase in demand for the commodity, and their decrease leads to a decrease in demand for the commodity.

8.2 Elasticity of demand: It expresses the extent to which a change in the required quantity of a particular commodity responds to a change that occurs in one of the factors affecting demand.

1.8.2 Price elasticity of demand: It is the degree of responsiveness of the quantity demanded of a commodity to the change that occurs in its price.

The goal of measuring it is for producers to know how to act regarding the commodity being studied in terms of production. If the demand is inelastic, producers can raise the price, while if the demand for the commodity is elastic, it is preferable to reduce the price of the commodity. It is measured by dividing the relative change in the quantity demanded of the commodity studied by the relative change in its price.

$$E_P = \frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q / Q}{\Delta P / P} = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

In the case of continuous data, that is, those written in the form of a demand function, the derivative is used as an approximation of the ratio of the change in the quantity demanded to the change in its price, and the elasticity relationship is written as follows:

$$E_P = \frac{\partial Q}{\partial P} \times \frac{P}{Q}$$

Degrees of elasticity: The price elasticity of demand is negative to indicate the inverse relationship between the quantity demanded and the price. To identify the type of demand, we compare the value of the price elasticity of demand taken in absolute value with the following cases:

1. If $E_P = 0$ the demand is inelastic: which means that any change in the price of the commodity does not lead to a change in the quantity demanded of it, such as the demand for medicines. A decrease in the price of the medicine by 70% does not lead to an increase in the quantity demanded.

2. If $0 < E_P < 1$ the demand is inelastic: This means that the percentage of change in the quantity demanded is less than the percentage of change in price.

3. If $E_P = 1$ the demand is isoelastic (perfectly elastic): that is, the percentage change in the quantity demanded is equivalent to the percentage change in price.

4. If $E_P > 1$ demand is elastic: then the percentage of change in the quantity demanded is greater than the percentage of change in price.

5. If $E_P = \infty$ demand is infinitely elastic: the degree of response of the quantity demanded to a change in price is very high, such as changes that occur in the stock market.

Note: In the case of one point, the price elasticity of demand can be calculated at that point and it is called the point elasticity. However, in the case of two points on the demand curve, the price elasticity between them can be calculated and it is called the arc elasticity, where the average of the two prices and the average of the two quantities are used as follows:

$$E_p = \frac{\Delta Q}{\Delta P} \times \frac{(P_1+P_2)/_2}{(Q_1+Q_2)/_2} = \frac{\Delta Q}{\Delta P} \times \frac{(P_1+P_2)}{(Q_1+Q_2)}$$

Example

If a decrease in the price of table salt by 50% increases the quantity demanded of it by 10%, calculate the price elasticity of demand for salt.

The solution

Our price elasticity of demand is the proportional change of quantity over the proportional change in price

$$E_P = \frac{\%\Delta Q}{\%\Delta P} = \frac{10}{-5} = -0{,}2$$

Since the absolute value of the price elasticity of demand is limited to zero and one, the demand for salt is inelastic. The value of elasticity means that the required quantity of salt increases by 0.2% if its price decreases by 1%. The negative sign indicates the inverse relationship between the price and quantity of salt.

2.8.2 Cross elasticity of demand: It is the degree of response of demand for a commodity to the relative change in the prices of other commodities (substitute and complementary).

If X is the commodity studied and Y is another commodity, the cross elasticity between them is written as follows:

$$E_{xy} = \frac{\partial Q_x}{\partial P_y} \times \frac{P_y}{Q_x}$$

Note:

- If $E_{xy} < 0$, then the two goods are complementary

- If $0 < E_{xy}$, then the two goods are substitutes
- If $E_{xy} = 0$, then the two goods are independent

Example

Car fuel prices increased from 0.1 dinars to 0.18 dinars per liter, and the demand for private cars decreased from 100,000 to 70,000 cars annually. Calculate the cross elasticity of demand between cars and fuel.

The solution

We've got

$$E_{xy} = \frac{\Delta Q_x}{\Delta P_y} \times \frac{P_y}{Q_x} = \frac{Q_{x2} - Q_{X1}}{P_{y2} - P_{y1}} \frac{P_{y1}}{Q_{x1}}$$

$$E_{xy} = \frac{70000 - 100000}{0.18 - \ .1} \times \frac{0.1}{100000} = -0{,}375$$

Since the cross elasticity of demand is negative, the two goods are complementary, and the value of the elasticity means that the demand for cars decreases by 0.375% if the fuel price increases by 1%.

3.8.2 Income elasticity of demand: It is the degree of responsiveness of demand for a good to the relative change in income.

It can be calculated using the following relationship:

$$E_P = \frac{\partial Q}{\partial R} \times \frac{R}{Q}$$

Since the relationship between demand and income is direct, that is, the quantity demanded changes in the same direction as income changes, the sign of income elasticity of demand is positive, except in the case of inferior goods, where it is negative, due to the inverse relationship between the quantity demanded and income.

Note:

- If $E_R < 0$, then the commodity is inferior
- If $0 < E_R < 1$, then the commodity is necessary
- If $1 < E_R$, then the commodity is a luxury

Example

If the ratio of the change in quantity demanded to the change in income is 0.75, calculate the income elasticity of demand for air travel tickets when income is 400 dinars and quantity demanded is 6 tickets.

The solution

We've got:

$$E_P = \frac{\Delta Q}{\Delta R} \times \frac{R}{Q} = 0{,}75 \frac{400}{6} = 50$$

Since the income elasticity of demand is greater than one, this good is a luxury good.

2-8-4 Factors affecting price elasticity of demand

4-1) Abundance of substitutes: The more substitutes there are for a good, the more elastic the demand for it is, and the fewer the substitutes, the less elastic the demand for the good is.

4-2) Necessity of the commodity: If the commodity is necessary, the demand for it will be slightly elastic, but if it is a luxury, the demand for it will be elastic.

4-3) The proportion of income spent on the commodity: The greater the proportion of income spent on the commodity, the more elastic the demand for it is, and the lower the demand, the less elastic the demand.

4-4) Multiple uses of the good: If the good has multiple uses, the demand for it is elastic, and if it is single-use, the demand for it is inelastic.

5.8.2 The relationship of price elasticity of demand to total revenue

Total Revenue is the total value of an organization's sales during a certain period. It is the product of multiplying the number of units sold by the price of one unit, that is:

$TR = P \times Q$.

The organization aims to maximize profit, which is represented by the difference between total revenue and total costs. To achieve this, the organization works to reduce costs, but if it exhausts its ability to reduce costs, it tends to follow a pricing policy that supports increasing revenue and thus increasing profit, but since total revenue is The result of multiplying quantity by price, and because of the inverse relationship between these two latter, an increase in price leads to a decrease in quantity, and therefore the effect on total revenue is uncertain, and depends on the extent to which quantity changes in relation to the change in price, that is, it depends on the price elasticity of demand.

The change in price and the quantity sold are two forces that work in two different directions, and total revenue is the result that follows the direction of the greater force. In the case of elastic demand, the percentage of change in quantity is greater than the percentage of change in price, and therefore total revenue in this case follows the direction of change in quantity as the force. The largest, that is, if the quantity decreases due to the high price, the revenue decreases, and if the quantity increases due to the low price, the revenue increases (the relationship between revenue and price is inverse). In the case of inelastic demand, the percentage of change in quantity is less than the percentage of change in price, and therefore total revenue follows the price in this case. If the price rises, revenue increases, and if it decreases, revenue decreases (the relationship between revenue and price is direct). In the case of isoelastic demand, where the percentage change in quantity is equal to the percentage change in price, the total revenue remains constant and is not affected. The following table summarizes the three cases as follows:

Table (2-2): The relationship between price elasticity and total revenue

	The effect of the change in price on total revenue		
Change in price	Flexible demand	Equi-elastic demand	Flexible demand
increase	increase	Not affected	decrease
decrease	decrease	Not affected	increase

From the table it is clear that it is in the producer's interest to increase the price in the case of inelastic demand for his commodity, and to reduce it in the case of elastic demand for it.

3 .Supply theory

1.3 Definition of supply: It is the different quantities of a commodity that producers are willing and able to supply to the market, at different prices and during a specific period of time, with the rest of the determinants of supply remaining constant.

2.3 The Law of Supply: This law states that there is a direct relationship between the quantity supplied of a certain commodity and its price, with the rest of the determinants of supply remaining constant. That is, the higher the price of the commodity, the higher the quantity supplied, and the lower its price, the lower the quantity supplied.

This direct relationship between the quantity and price of a good can be expressed either as a supply schedule, supply curve, or supply function.

3.3 Supply table: It is a table that shows the various quantities that the producer offers at different possible prices for the commodity (Table 3-1).

Table (3-1): Supply Schedule

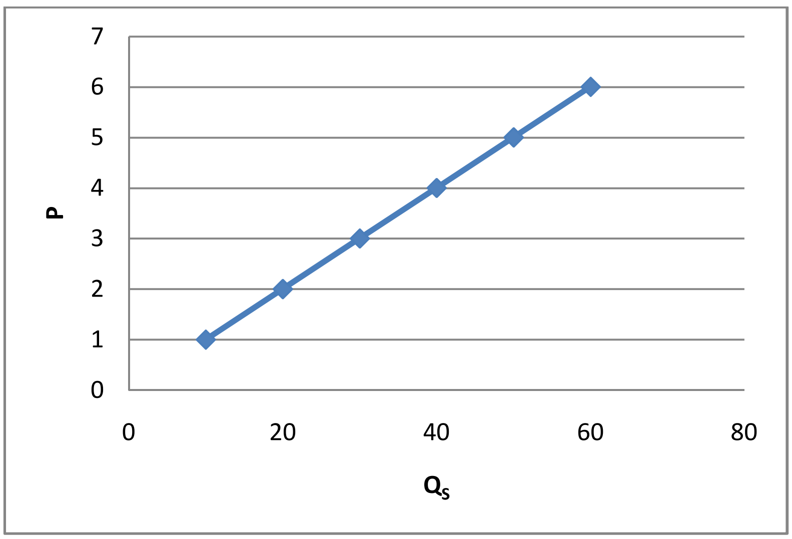
Price P (dinar)	Quantity supplied Q (kg)
1	10
2	20
3	30
4	40
5	50
6	60

We notice from the table that the law of supply is embodied, where an increase in the price, for example, from 2 to 3 dinars leads to an increase in the quantity supplied from 20 to 30 kg.

Note: A market supply table is a table that includes the sum of all individual offers at their corresponding prices in the market for a given commodity.

4.3 Supply curve: It is a curve that shows the direct relationship between the various quantities supplied of a particular commodity, which are represented on the interval axis, and the corresponding potential prices, which are represented on the ranking axis (Figure 3-1).

Figure (3-1): Individual supply curve



This curve represents the data of the supply table, and the supply curve is characterized by a rise from the bottom left to the top right, that is, it has a positive slope, because there is a direct relationship between the quantity supplied and the price, and it is not required to be linear as in this case, it can be non-linear.

5.3 Supply function: It is a function that links the quantity supplied of a particular commodity as a dependent variable, with the most important factors affecting it as independent variables, the latter of which are called the determinants of supply.

$$Q_s = f(P_x, P_y, P_L, P_K, \ldots, P_T)$$

Where: P_x : the price of the commodity studied, P_y: the prices of other commodities, P_L, P_K: the prices of production factors, P_T: the value of the technical level of production.

6.3 Exceptions to the law of supply

The law of supply may not be achieved in some cases in real life, as a rise in price leads to an increase in the quantity supplied, and a decrease in it leads to a decrease in the quantity supplied, and these cases include:

1) Expecting a continued decrease or increase in the price of the commodity: When producers expect a continued increase in price, they prefer not to respond, but rather reduce the supply of their goods in order to achieve greater profits when the price reaches its maximum, and the opposite happens when they expect the price to continue to decrease.

2) The supply of agricultural crops is relatively fixed: Since the agricultural area is limited, and there is a period of time between sowing and harvesting the crop, if the time for harvesting the crop begins and prices rise, the farmer cannot increase the cultivated areas, and lower prices do not lead to a contraction in the supply of agricultural crops.

3) The job offer often contradicts the labor law: the worker is sometimes forced to increase working hours when his real wage decreases, and also reduce working hours if his real wage increases.

7.3 Display parameters

The quantities offered of a good or service during a certain period of time depend on several determinants, the most important of which are:

1) Price of the commodity: If the rest of the factors affecting the supply other than the price remain constant, there will be a direct relationship between the quantities supplied of the commodity and its price, whereby the higher the price, the greater the producer's profits, which prompts him to increase the quantity supplied, and the opposite occurs when the price decreases.

2) Prices of other goods

Commodities are divided according to their supply being affected by price movements into:

a. Substitute goods in production: They are goods in which one of them can replace the other in production. If the price of one of them increases, the quantity produced of it increases, while the quantity produced of the commodity whose price remains constant, such as wheat and corn, decreases, and because the producer is looking for the highest possible profit. Thus, it tends to produce alternative goods whose price has become relatively higher, and decreases the production of the good whose price remains constant.

B. Integrated goods in production: They are goods in which one cannot be produced without the other, so their production corresponds with an increase or decrease, such as meat and leather. An increase in the price of the complementary good leads to an increase in its supply. This increase in supply leads to an increase in the supply of the studied good despite its constant price.

3) Prices of production factors: There is an inverse relationship between the quantity supplied of a commodity and the prices of factors of production, which are considered costs for the producer. The higher their prices, the greater the production costs, which leads to a decrease in the supply of the commodity, while the supply of the commodity increases if the prices of production factors decrease. .

4) Technical level of production: There is a direct relationship between the quantity supplied of a commodity and the technical level of production. The greater the technological progress in producing a commodity, the lower the production costs and thus the greater the supply of the commodity.

5) Taxes imposed by the government: Imposing taxes leads to raising production expenses, which leads producers to reduce the supply of the commodity, while reducing taxes reduces expenses and leads to increased production.

6) Subsidies provided by the government: Providing a subsidy to support a specific commodity by the government leads to an increase in the quantity supplied, while reducing the subsidy for it leads to a reduction in the quantity produced and therefore the quantity supplied.

8.3 Change in quantity supplied and change in supply

The change in the quantity supplied arises as a result of a change in the price of the commodity itself, and is achieved graphically by moving along the supply curve, upward when the price rises and downward when the price falls. As for the change in supply, it results from a change in one of the factors affecting it other than the price of the commodity studied, and it causes the entire supply curve to shift to the left in the case of a decrease in supply, and to the right in the case of an increase in supply.

9.3 Elasticity of supply: It expresses the extent to which a change in the quantity supplied of a particular commodity responds to a change that occurs in one of the factors affecting supply.

1.9.3 Price elasticity of supply: It is the degree of responsiveness of the quantity supplied of a commodity to the change that occurs in its price, with the rest of the determinants of supply remaining constant, and it is always positive.

It is measured by dividing the relative change in the quantity supplied of the commodity studied by the relative change in its price.

$$\mathrm{E_P} = \frac{\%\Delta \mathrm{Q}}{\%\Delta \mathrm{P}} = \frac{\Delta Q / \mathrm{Q}}{\Delta \mathrm{P} / \mathrm{P}} = \frac{\Delta \mathrm{Q}}{\Delta \mathrm{P}} \times \frac{\mathrm{P}}{\mathrm{Q}}$$

In the case of continuous data, that is, those written in the form of a supply function, the derivative is used as an approximation of the ratio of the change in the quantity supplied to the change in its price, and the elasticity relationship is written as follows:

$$E_P = \frac{\partial Q}{\partial P} \times \frac{P}{Q}$$

Degrees of elasticity: The price elasticity of supply is positive to indicate the direct relationship between the quantity supplied and the price. To identify the type of supply, we compare the value of the price elasticity of supply, taken in absolute value, in the following cases:

1. If $E_P = 0$ the supply is inelastic: which means that any change in the price of the good does not lead to a change in the quantity supplied.

2. If $0 < E_P < 1$ the supply is inelastic (inelastic): this means that the percentage change in the quantity supplied is less than the percentage change in price.

3. If $\mathrm{E}_P = 1$ the supply is isoelastic (perfectly elastic): that is, the percentage change in the quantity supplied is equivalent to the percentage change in price.

4. If $\mathrm{E_P} > 1$ supply is elastic: then the percentage of change in the quantity supplied will be greater than the percentage of change in price.

5. If $\mathrm{E}_P = \infty$ supply is infinitely elastic: the degree of response of the quantity supplied to a change in price is very high.

Example

Let the supply function of good x be the following: $Q_{sx} = 10 + 3P_x$

Calculate the price elasticity of supply when P=5.

The solution

We've got: $E_P = \frac{\partial Q}{\partial P} \times \frac{P}{Q}$

$$E_P = 3 \times \frac{5}{10+3(5)} = 0.6$$

Since the price elasticity of supply for commodity x is limited to zero and one, the supply of this commodity is inelastic, and its value means that the quantity changes by 0.6% if the price changes by 1% .

2.9.3 Factors affecting elasticity of supply

Storability: The more a commodity is storable and has a reasonable cost, the more elastic its supply is to changes in price, and the more the commodity is unstorable and perishable, the more inelastic its supply is.

Portability: When a commodity is transportable and at reasonable costs, its flexibility is greater. If the price of a commodity falls in one region and it is transportable, the producer transports it and sells it in another region where prices have not fallen.

The nature of the production process: The more it is possible to change the volume of production at lower costs and in an easier way, the more flexible the supply of the commodity will be, and the ease of changing production factors increases the elasticity of the commodity, and vice versa.

Future Price Expectations: If the forecast suggests that the current rise in prices will continue, supply will be more elastic than if the forecast suggests a temporary rise followed by a decline in prices.

Time period: The longer the time period, the more elastic the supply is, but in the short period, it is inelastic.

4 .Equilibrium

Consumers represent the demand side in the market for goods and services, and the demand table reflects their behavior and desires. Producers also represent the supply side, and the supply table reflects their behavior and decisions. We also pointed out that completing the exchange process in the market cannot take place unless both parties agree on two important elements that represent the basis of the exchange process. They are the price of the commodity and the quantity that will be exchanged. But who sets the price? If the producer is given the freedom to determine the price, he will want the highest possible price to give away the commodity, but if he leaves it to the consumer to determine the price, he will want to pay the lowest possible price to obtain the commodity. Here the difference in behavior and desires appears. Therefore, it was necessary to have a consensus price that would be satisfied by producers and consumers alike. This is expressed as the equilibrium price, and a specific quantity is exchanged, called the equilibrium quantity.

1.4 Equilibrium price

It is the price at which the quantity that consumers are willing to buy of a good or service is equal to the quantity that sellers are willing to sell it, and that quantity is called the equilibrium quantity.

2.4 Determination of the equilibrium point

A) Graphically: Equilibrium is determined graphically by the intersection of the demand curve with the supply curve, where the intersection point represents the equilibrium point.

Example: The following table shows the market demand and supply for a particular commodity.

price	Required quantity	Quantity offered	difference	Market situation
10	50	270	220	surplus
8	100	210	110	surplus
6	150	150	0	equilibrum
4	200	90	-110	inability
2	250	30	-220	inability

We note from the previous table the following:

The equilibrium price at which the quantity demanded equals the quantity supplied is 6, and at this point the market does not witness any surplus or deficit. It is noted from the table that the quantity demanded is equal to 150 units and so is the quantity supplied.

At price levels higher than the equilibrium price, the market is in a state of surplus, that is, the quantity supplied is greater than the quantity demanded. For example, at price 8, the quantity demanded is 100 units and the quantity supplied is 210 units, so the surplus is 110 units.

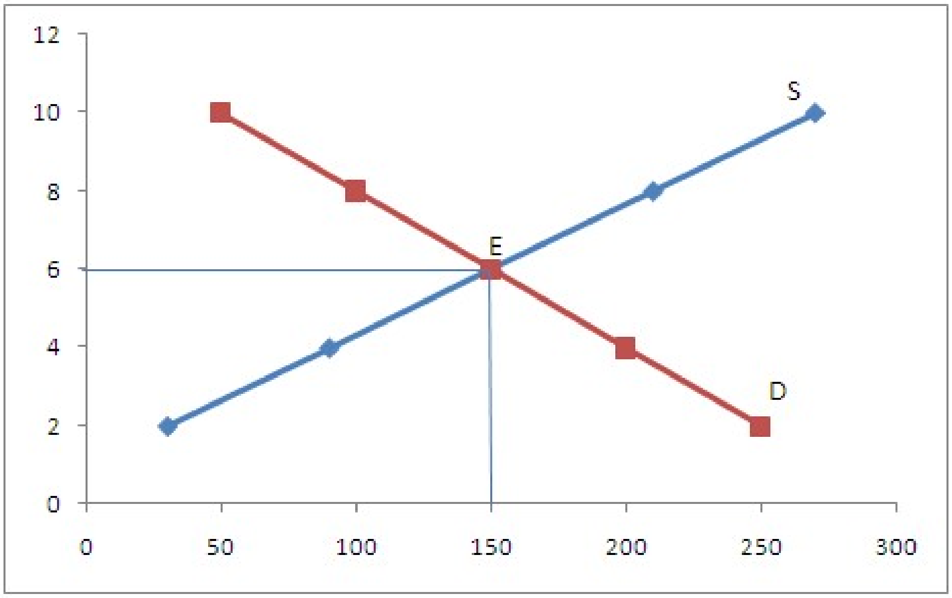
The presence of a surplus in the market puts downward pressure on the price, as producers are willing to accept a lower price to get rid of the surplus, which returns the market to equilibrium.

At price levels lower than the equilibrium price, the market is in a state of deficit, meaning that the quantity supplied is less than the quantity demanded. For example, at price 4, the quantity demanded is 200 units and the quantity supplied is 90 units, so the deficit is 110 units.

A market deficit puts upward pressure on the price, as consumers are willing to pay a higher price to obtain the good, which returns the market to equilibrium.

By representing both the market demand schedule and the market supply schedule graphically, we obtain the equilibrium point E, which is the point of intersection of the demand curve D with the supply curve S.

Figure (4-1): A figure that graphically shows the equilibrium point



b) Mathematically

The equilibrium point can be obtained through the demand and supply functions, by achieving the equilibrium condition $(Q_D = Q_S)$, where:

$$Q_D = a - bp, \qquad b > 0 \qquad \text{Demand function}$$

$$Q_S = c + dp, \qquad d > 0 \qquad \text{Supply function}$$

Example: Let the following be the demand and supply equations for a particular good:

1) Determine the equilibrium price and quantity

2) What is the market condition at price 8?

3) What is the price at which the market witnesses a surplus of 40 units?

The solution

1) Determination of equilibrium price and quantity: at equilibrium the quantity demanded equal quantity supplied, and then we find value price equilibrium, Then we make up in any from the two equations to find quantity equilibrium.

$$Q_d = Q_s$$

$$\Rightarrow 60 - 4p = 6p - 40$$

$$\Rightarrow 100 = 10p \;\Rightarrow\quad p^* = 10,\; Q^* = 20$$

2) Market situation at price 8: To know the market situation, we must find the quantity demanded and the quantity supplied at this price by substituting in the demand equation and the supply equation.

$$Q_d = 60 - 4p = 60 - 4 \times 8 = 28$$

$$Q_s = 6p - 40 = 6 \times 8 - 40 = 8$$

Since the quantity demanded is greater than the quantity supplied, the market is in a deficit of 20 units, and this is normal because the price 8 is less than the equilibrium price.

3) The value of the price in the case of a surplus of 40 units: The surplus is calculated by subtracting the demanded quantity from the supplied quantity, substituting the value of the surplus, and then searching for the value of the price that fulfills the equation as follows:

$$surplus = Q_s - Q_d$$
$$40 = 6p - 40 - (60 - 4p)$$
$$p = 14$$
$$Q_d = 60 - 56 = 4$$
$$Q_s = 84 - 40 = 44$$

Thus, the price that leads to a surplus of 40 units is 14, and this was confirmed through compensation in the demand equation and the supply equation.

3.4 Change in market equilibrium

We pointed out during the discussion about the difference between the change in demand and the change in the quantity required and the difference between the change in supply and the change in the quantity supplied, that the quantity demanded and the quantity supplied change if the price of the commodity changes, and the direction of movement is according to the law of demand and the law of supply. As for demand and supply, they change if one or more of the factors determining each of them change. We also pointed out that the change in both demand and supply can be represented graphically in the form of a shift of the demand curve and the supply curve, respectively. From the above, we can say that both the location of the demand curve and the location of the supply curve are subject to change, and thus the equilibrium point is also subject to change, and then both the equilibrium price and the equilibrium quantity change. But how does a change in both demand and supply affect the equilibrium price and quantity?

The following table shows cases of change in both demand and supply and their impact on the equilibrium price.

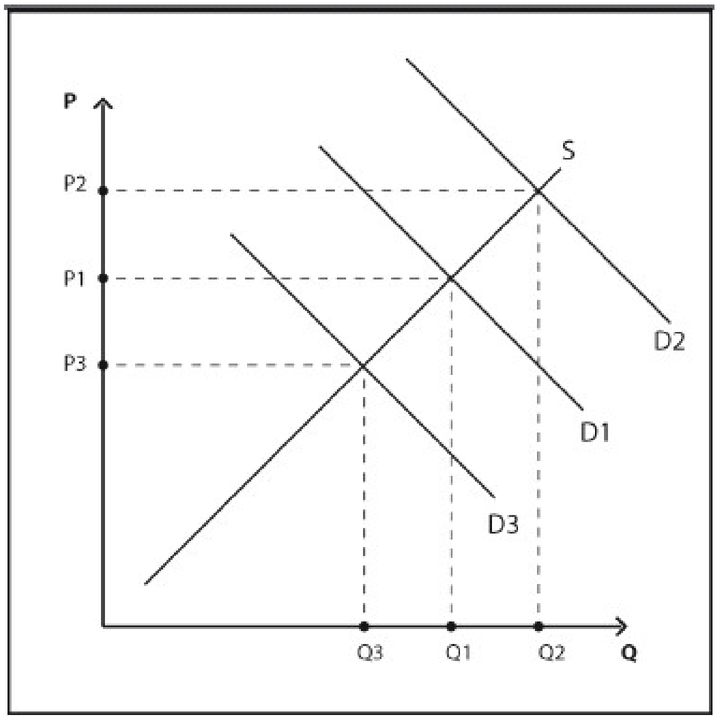
Table (4-1): The effect of changing demand and supply on the equilibrium point

The change	Graphical representation	Equilibrium price	Equilibrium quantity
Demand + Supply =	The demand curve shifts to the right and the supply curve remains constant	+	+
Demand - Supply =	The demand curve shifts to the left and the supply curve remains constant	-	-
Demand = Supply +	The supply curve shifts to the right and the demand curve remains constant	-	+
Demand = Supply -	The supply curve shifts to the left and the demand curve remains constant	+	-
Demand + Supply +	Both the demand and supply curves shift to the right	+ or - or =	+
Demand - Supply -	Both the demand and supply curves shift to the left	+ or - or =	-
Demand + Supply -	The demand curve shifts to the right and the supply curve shifts to the left	+	+ or - or =
Demand - Supply +	The demand curve shifts to the left and the supply curve shifts to the right	-	+ or - or =

It is noted from the table that:

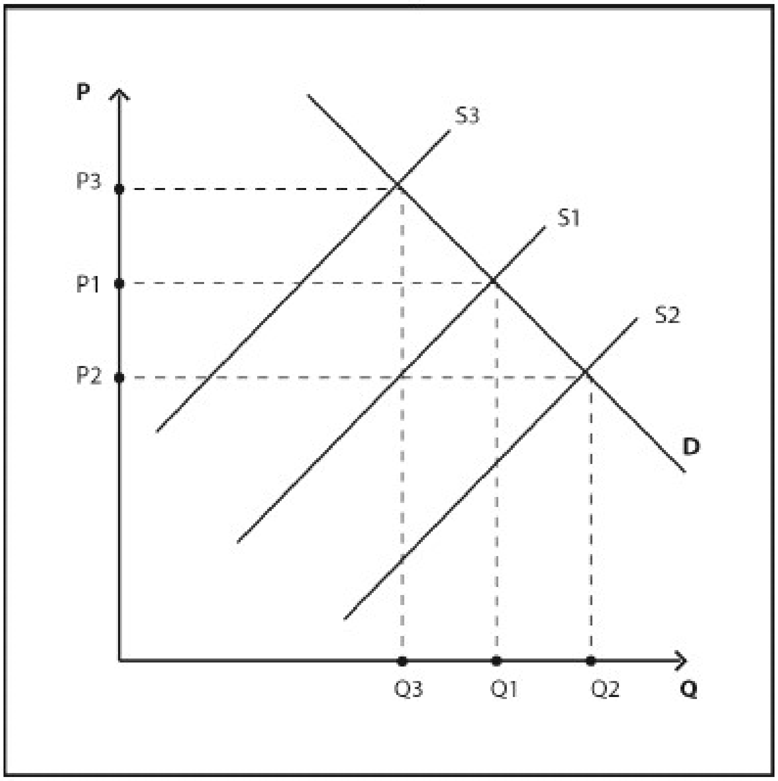
An increase in demand, with other factors remaining constant, leads to an increase in price as demand and competition for the same quantity supplied increases, causing upward pressure on the price, and vice versa.

Figure (4-2): Market equilibrium in the case of a change in demand



An increase in supply, with other factors remaining constant, leads to a decrease in price, because the quantities supplied increase at all prices while the quantity demanded remains constant at the same prices, which leads to downward pressure on the price.

Figure (4-3): Market equilibrium in the case of a change in supply



An increase in demand and an increase in supply together leads to an increase in the equilibrium quantity because the quantities demanded and the quantities supplied will increase, but the change in the equilibrium price depends on the size of the change in demand and the size of the change in supply. If the change in demand is greater (demand has increased at a rate higher than the rise in supply), then The equilibrium price will rise and vice versa. If the increase in demand equals the increase in supply, the price will not change. The same analysis applies when both demand and supply decrease together.

If demand rises and supply falls, the equilibrium price will inevitably rise because both changes lead to the same result. The equilibrium quantity depends on the size of the increase in demand and the size of the decrease in supply. If the increase in demand is greater than the decrease in supply, the equilibrium quantity will rise, and vice versa.

5 .Applications on equilibrium

1.5 The effect of taxes on competitive market equilibrium

We previously pointed out that taxes are one of the most important determinants of supply. When a tax is imposed on a product, production costs increase and the supply curve shifts to the left, which leads to an increase in the equilibrium price and a decrease in the equilibrium quantity.

The producer is the one who pays the tax to the government, but it is usually distributed in different proportions between the producer and the consumer, and the distribution ratio depends on the elasticity of both demand and supply. Taxes are classified into two types: specific production taxes and ad valorem production taxes.

1.1.5 Specific production taxes: They are the imposition of a specific amount on each productive unit. For example, an amount of 1 DZD is imposed on each productive unit. Its effect can be explained through a specific commodity market model as follows:

$Q_D = a - bp, \qquad b > 0$ Demand function

$Q_S = c + dp, \qquad d > 0$ Supply function

$a > c \qquad Q_D = Q_S$ Equilibrium equation

When a t specific tax is imposed, the demand function remains the same, while the supply function becomes:

$$Q_S' = c + d(p - t)$$

Finding the equilibrium price after imposing the tax:

$$Q_D = Q_S' \Rightarrow a - bp = c + d(p - t)$$

$$\Rightarrow a - c + dt = (b + d)t$$

$$\Rightarrow p^* = \frac{a-c+dt}{b+d}$$

$$\Rightarrow p^* = \frac{a-c}{b+d} + \frac{dt}{b+d}$$

We note that the difference between the equilibrium prices before and after imposing the tax is $\frac{dt}{b+d}$ (it is zero when no tax is imposed).

To know the effect of the specific tax on the equilibrium price, we calculate the derivative: $\frac{\partial p^*}{\partial \mathrm{t}}$

$\frac{\partial p^*}{\partial \mathrm{t}} = \frac{d}{b+d} > 0$ Because $b > 0$ *and* $d > 0$

Since $b + d > d$ then $1 > \frac{d}{b+d} > 0$

We note that the specific tax affects the equilibrium price upward, but by an amount less than the tax rate.

Finding the equilibrium quantity after imposing the tax:

Substituting the new equilibrium price into the demand function we find:

$$\mathrm{Q_D} = a - bp^* = a - b\left(\frac{a-c}{b+d} + \frac{dt}{b+d}\right)$$

$$\Longrightarrow Q^* = \frac{ad+bc}{b+d} - \frac{bd}{b+d}t$$

We note that the difference between the equilibrium quantity before the tax was imposed and after it was imposed is $\frac{bd}{b+d}t$ (it is zero if no tax is imposed). This is the same amount by which the equilibrium quantity decreases.

Distribution of the tax burden

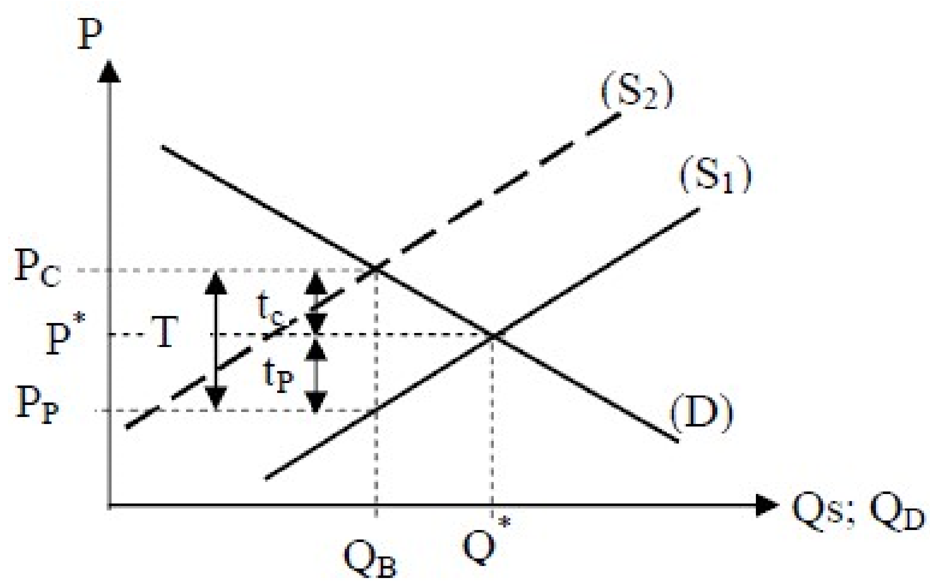
When the government impose tax on a particular commodity, the party of consumer and producer that its elasticity price is Larger, it will bear a burden less from the other party, and vice versa correct. This is done according to the following details:

- The proportion of the tax that the producer bears increases as the price elasticity of supply for the commodity decreases. Therefore, the producer bears the full tax when supply is inelastic.

- If the price elasticity of demand is greater than the price elasticity of supply, then the tax the consumer bears is less than what the producer bears, and vice versa.
- If the price elasticity of demand is less than the price elasticity of supply, then the tax borne by the consumer is greater than what is borne by the producer.
- If the price elasticity of demand and the price elasticity of supply are equal, the producer and consumer share the tax burden equally.

The effect of the tax on market equilibrium, as well as the distribution of the tax burden, can be illustrated through the following figure:

Figure 5-1 : The effect of the tax on market equilibrium



We note from the figure that the equilibrium point before imposing the tax is $A(p^{*}, Q^{*})$, but after imposing the tax, the equilibrium point changed due to the shift of the supply curve to the left, and two prices appeared, the difference between which expresses the amount of the tax as follows: $T = p_c - p_p$, and the tax amount can also be found by the following relationship:

$$T = t_c + t_p$$

where:

p_c: Represent the price that He accepts Paid the buyer to get on Item after duty Tax

p_p: Represent the price that He picks it up the seller after duty Tax

T: represents the amount of tax

t_c: The amount of tax the consumer bears ($t_c = p_c - p^*$)

t_p: Tax borne by the seller ($t_p = p^* - p_p$) or ($t_p = T - t_c$)

Example: Let the demand and supply equations for a particular commodity be as follows:

$$p = 40 - 4Q_d \quad , \quad p = 2Q_s + 4$$

If the government imposes a specific tax on the product of 6 DZD, explain how the tax burden is distributed between the consumer and the producer.

The solution

1) Finding the equilibrium point before imposing the tax:

$$p = p \Rightarrow 40 - 4Q = 2Q + 4$$

$$\Rightarrow 36 = 6Q \Rightarrow Q^* = 6 \;\; and \;\; p^* = 16$$

2) Finding the equilibrium point after imposing the tax:

Imposing a tax on the product appears in the form of a decrease in the price that the producer receives, and then the supply equation changes to become as follows:

$$(p - 6) = 2Q_s + 4$$

Hence, the new equilibrium point is as follows:

$$p = p \Rightarrow 40 - 4Q = 2Q + 10$$

$$\Rightarrow 30 = 6Q \Rightarrow Q^* = 5 \;\; \text{and} \;\; p^* = 20$$

We notice from the above that the price paid by the consumer increased from 16 DZD to 20 DZD, that is, by 4 DZD, while the producer bore the remainder of the tax, which is 2 DZD. Therefore, we say that the consumer bore a greater percentage of the tax because the price elasticity of demand was less than the price elasticity of supply at the original equilibrium price. This can be confirmed by calculating the elasticity at price 16.

Price elasticity of demand: $$E_d = \frac{\partial Q}{\partial P} \times \frac{P}{Q} = -\frac{1}{4} \times \frac{16}{6} = -\frac{2}{3}$$

Price elasticity of supply: $$E_s = \frac{\partial Q}{\partial P} \times \frac{P}{Q} = \frac{1}{2} \times \frac{16}{6} = \frac{4}{3}$$

1.2.5 Ad valorem production taxes : They are the imposition of a certain percentage on the price of each unit of production. If the tax percentage is r, the price becomes as follows: $p^r = p(1-r)$

The supply function after imposing the ad valorem tax becomes:

$$Q_S' = c + dp^r = c + dp(1-r)$$

$$\Rightarrow \ Q_S' = c + dp - dpr$$

So the commodity market model is:

$$\mathrm{Q_d} = a - bp$$

$$Q_S' = \mathrm{c + dp - dpr}$$

$$Q_d = Q_s$$

By solving the model, we find the equilibrium price and equilibrium quantity, respectively:

$$p^* = \frac{a-c}{d+b-dr} \qquad , \qquad Q^* = \frac{ad+bc-ad}{b+d-d}$$

To know the effect of the ad valorem tax on the equilibrium quantity, we derive the latter with respect to the tax, and we find that it leads to a decrease in the equilibrium quantity.

Example: Let the demand and supply functions for a particular commodity be as follows:

$$\mathrm{Q_d} = 20 - \frac{21}{4}p$$

$$Q_s = 2 + p$$

Show the effect of imposing an ad valorem tax of 25 % on the price of the unit sold on the equilibrium point.

The solution

- Finding the equilibrium point before imposing the tax

$$Q_d = 20 - \frac{21}{4}p \quad \text{and} \qquad Q_s = 2 + p$$

$$20 - \frac{21}{4}p = 2 + p \Longrightarrow Q_d = Q_s$$

$$\Longrightarrow \frac{25}{4}p = 18$$

$$\Longrightarrow p^* = \frac{72}{25}$$

Substituting in Q_s, we find:

$$Q^* = \frac{122}{25}$$

- Finding the equilibrium point after imposing the tax

$$\mathrm{Q_d} = 20 - \frac{21}{4}\mathrm{p} \quad \text{and} \qquad \mathrm{Q'_S} = 2 + \mathrm{p}(1 - \frac{1}{4})$$

$$20 - \frac{21}{4}\mathrm{p} = 2 + \mathrm{p}(1 - \frac{1}{4}) \Longrightarrow \mathrm{Q_d} = \mathrm{Q'_S}$$

$$\Rightarrow 6\mathrm{p} = 18$$

$$\Rightarrow \mathrm{p}^* = 3$$

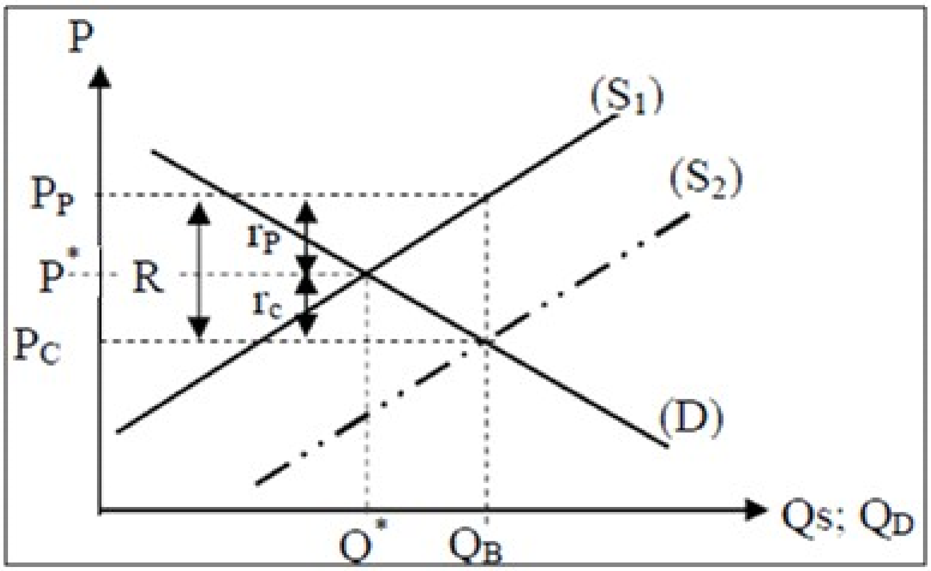
Substituting in Q_d, we find:

$$\mathrm{Q}^* = \frac{17}{4}$$

2.5 The effect of subsidies on competitive market equilibrium

The government provides subsidies to producers in the event of a need to achieve an increase in the supply of a particular commodity. Therefore, these subsidies can be considered as a negative tax that is added to the price instead of subtracted from it, and its effect on the equilibrium point appears graphically, as the following figure shows:

Figure (5-2): The effect of the subsidy on the equilibrium point



- Finding the equilibrium values after providing the subsidy: To find the equilibrium values after providing the subsidy (S), the following set of equations is solved:

$Q_D = a - bp, \quad b > 0$ Demand function

$Q'_S = c + d(p + s), \quad d > 0$ New supply function

$Q_D = Q'_S \quad , \quad a > c$ equilibrium equation

After solving the set of equations, we obtain the equilibrium price and equilibrium quantity, respectively:

$$p^* = \frac{a-c}{d+b} - \frac{d}{d+b}t \qquad , \qquad Q^* = \frac{ad+b}{b+d} + \frac{bd}{b+d}t$$

The effect of providing a subsidy is that the equilibrium price decreases and the equilibrium quantity increases.

Determine of the amount of the subsidy

The amount of the subsidy is the difference between the price received by the seller P_p and the price paid by the consumer P_c. It can be found by the following relationship:

$$S = s_p + s_C$$

Where:

s_C: The amount the consumer benefits from the subsidy, which is calculated through the relationship $(s_c = P^* - P_c)$

s_p: The amount the producer benefits from the subsidy, which is calculated through the relationship $(s_p = P_p - P^*)$

Note: The consumer price P_c is the same as the equilibrium price after the subsidy is provided.

Example

Let the demand and supply functions for a particular commodity market be as follows:

$$Q_d = 10 - P \qquad\qquad Q_s = 2P - 5$$

1 - Calculate the market equilibrium price and quantity.

2 - The state grants a subsidy of 3 Dinars. What is the new equilibrium price and quantity? What is the amount of the subsidy that benefits both the producer and the consumer?

The solution

- Determine the balance point before granting the subsidy

$$\mathrm{Q_d} = \mathrm{Q_s} \Longrightarrow 10 - \mathrm{P} = 2\mathrm{P} - 5$$

$$\Longrightarrow \mathrm{P}^* = 5$$

Substituting P^* into the demand function we find: $\mathrm{Q}^* = 5$

- Determine the equilibrium point after granting the subsidy

The demand function remains the same, while the supply function becomes as follows:

$$Q_S' = 2(P+3) - 5$$

From the equilibrium condition:

$$Q_d = Q_s \Longrightarrow \; 10 - P = 2(P+3) - 5$$

$$\Longrightarrow \; \mathrm{P}_1^* = 3$$

Substituting P^* into the demand function we find: $\mathrm{Q}^* = 7$

- Determine consumer and producer prices

Substituting the new equilibrium quantity into the demand function we obtain the price paid by the consumer:

$$\mathrm{Q_d} = 10 - \mathrm{P} \; \Longrightarrow \mathrm{P} = 10 - \mathrm{Q_d}$$

$$\Longrightarrow \mathrm{P} = 10 - 7 = 3$$

The consumer price is $P_c = 3$, it is the same as the equilibrium price after providing the subsidy.

Substituting the new equilibrium quantity into the original supply function (before the subsidy was introduced) we obtain the price received by the producer:

$$\mathrm{Q_s} = 2\mathrm{P} - 5 \; \Longrightarrow \mathrm{P} = (7+5)/2$$

$$\Longrightarrow \mathrm{P} = 6$$

- The amount of subsidy that the consumer benefits from

$$\mathrm{s_c} = \mathrm{P}^* - \mathrm{P_c}$$

$$= 5 - 3$$

$$= 2$$

- The amount of subsidy that the producer benefits from

$$s_p = P_p - P^*$$

$$= 6 - 5$$

$$= 1$$

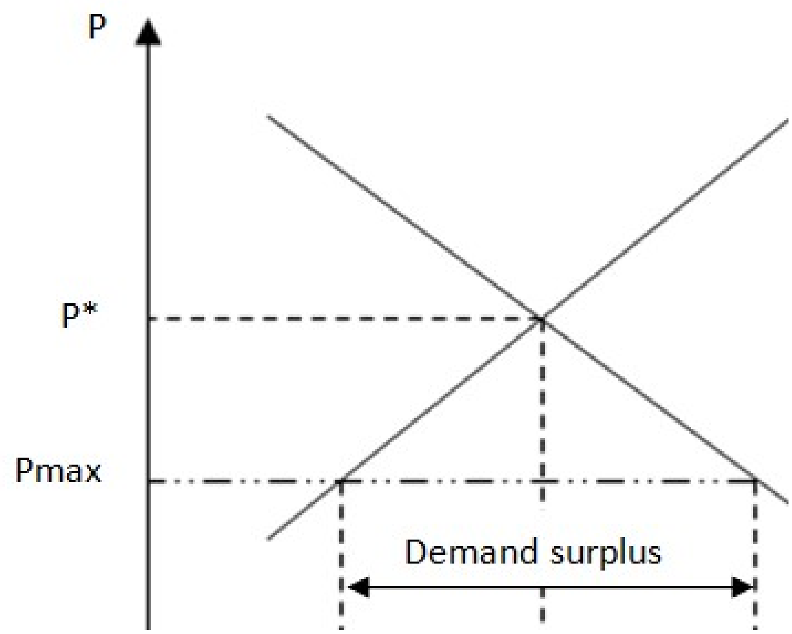
3.5 Price monitoring

Market mechanism indicates that the equilibrium price is determined by the interaction between market demand and market supply. It also indicates that the presence of a surplus at any price higher than the equilibrium price will lead to downward pressure on the price until the market returns to equilibrium. The presence of a deficit leads to upward pressure on the price until the market returns to equilibrium. But this is not the case when the government directly intervenes in the market mechanism with the aim of monitoring and controlling prices. The government may intervene through what is called the ceiling price or floor price.

1.3.5 Ceiling price

The ceiling price is defined as the price of a commodity that is less than the equilibrium price imposed by the government for the benefit of the consumer when the equilibrium price for this commodity is high, in order to ensure that the commodity reaches all consumers. An example of this is imposing a certain price for bread, and this government pricing policy leads to a deficit. In the market for this commodity, it also leads to the emergence of the black market, and the government takes several measures accompanying the ceiling price policy, the most important of which is supporting producers by reducing taxes or giving subsidies, and providing additional quantities of the commodity to fill the deficit in the market for that commodity by encouraging the import of good substitutes for the commodity.

Figure (5-3): The effect of the ceiling price on the equilibrium point



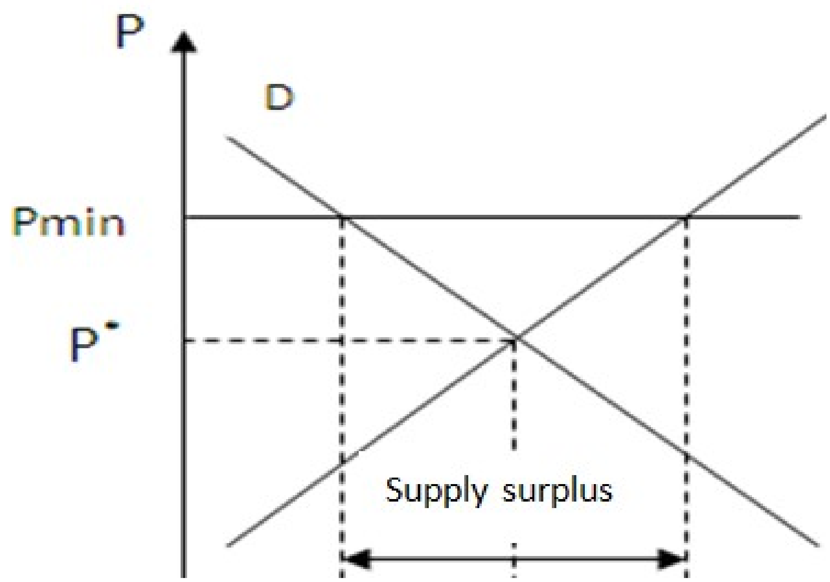
The equilibrium price and equilibrium quantity were P^* and Q^*, respectively, but after the government intervened by imposing a maximum price Pmax, the quantity supplied became Q_1 and the quantity demanded Q_2, and the difference between them ($\Delta Q=Q_2-Q_1$) represents surplus demand (or supply deficit).

2.3.5 Floor price

The floor price is defined as the price imposed by the government in favor of the producer, such that it is higher than the equilibrium price in order to ensure that producers continue to supply the market with a certain commodity. An example of this is setting a floor price for wheat in order to encourage producers of this strategic commodity to increase their production.

This pricing policy leads to a surplus in the commodity market, which may create additional burdens resulting from storing the commodity. The government takes several measures accompanying the floor price, the most important of which is protecting the local product and purchasing quantities that exceed consumers' needs.

Figure (5-4): The effect of the floor price on the equilibrium point



The equilibrium price and equilibrium quantity were P* and Q*, respectively, but after the government intervened by imposing a lower price Pmin, the quantity supplied became Q_s and the quantity demanded Q_d, and the difference between them ($\Delta Q = Q_s - Q_d$) represents the surplus supply (or the demand deficit).

Example:

Let the demand and supply functions for a particular good be as follows:

$$Q_d = 500 - 50P \quad , \qquad Q_s = 50P$$

We assume that the government imposes a floor price for this market of P=6.

- Find both the quantity demanded and the quantity supplied in this case
- Is this market in a case of surplus or in a case of deficit? How much is it?

The solution

Substituting the floor price into the demand function we obtain the quantity demanded

$$Q_d = 500 - 50(6) = 200$$

By substituting the floor price into the supply function, we obtain the quantity supplied

$$Q_s = 50(6) = 300$$

To know the market situation, we calculate the difference between the quantity demanded and the quantity supplied:

$$\Delta Q = \mathrm{Q_s} - Q_d = 300 - 200 = 100$$

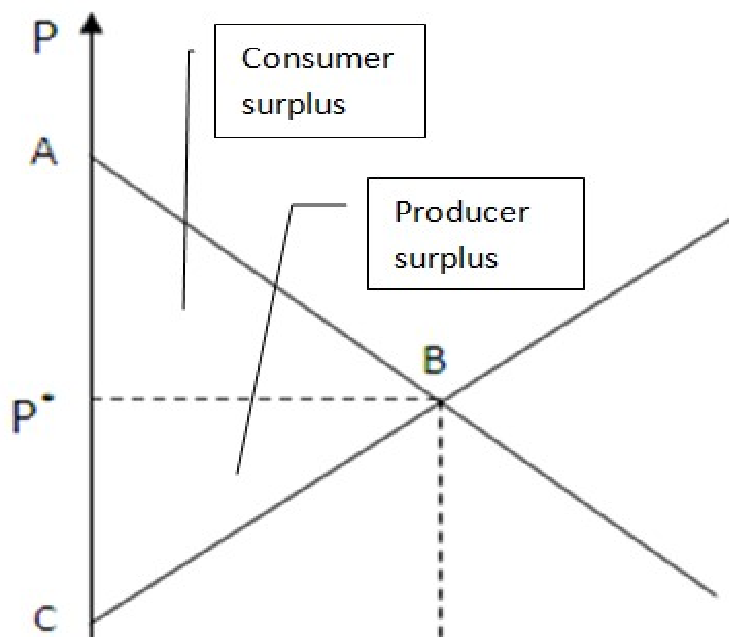
Since the quantity supplied is greater than the quantity demanded, the market is in a case of surplus.

4.5 Consumer surplus and producer surplus

Consumer surplus is the amount that the consumer keeps after purchasing a certain commodity. This amount is limited between the maximum price that the consumer is willing and able to pay to purchase this commodity and the market price (equilibrium price). As for the producer's surplus, it is an amount that the producer receives after selling a specific commodity, and it is limited to the market price and the lowest price that the producer accepts to sell his commodity.

Note: The amount is limited between two prices, which does not mean that the surplus is equal to the difference between these two prices, but rather the consumer surplus is determined The area of the triangle (ABP*) , and the producer surplus is determined by the area of the triangle (P*BC), as the following figure shows:

Figure 5-5: Consumer surplus and producer surplus



6 .Consumer behavior analysis

Utility theory is considered the basis on which the explanation of consumer behavior and interaction with economic variables is built, and on the basis of which he makes his decisions to choose between alternatives competing for his spending capabilities, i.e. monetary income and prices . The basic hypothesis of this theory is that consumption of goods and services generates benefit or satisfaction for the consumer, and consumer choices are determined and influenced by several factors, summed up in two highly important concepts: consumption possibilities and consumer preferences.

1.6 Consumption possibilities

Consumers' choices are made within the context of income and the prevailing price level of the goods and services they wish to purchase. During any period of time, the consumer has a specific amount of disposable income. He also faces the prevailing market prices for the goods and services that he wishes to consume, which he takes without the ability to influence them, in light of perfect competition. Income and price levels represent the constraints that govern the consumer's choices.

2.6 Consumer preferences

The consumer's choices depend on his preferences. He chooses according to what he likes and what he does not like. Economists use the concept of utility to describe and explain the consumer's preferences. They apply the term utility to the satisfaction or pleasure that a person gets from consuming a particular commodity.

3.6 **Utility theory**

The consumer's desire to obtain goods and services reflects his feeling of benefit, satisfaction, or material and moral satisfaction from consuming goods and services. For example, the consumer wants food, drink, and clothing because he obtains material and moral satisfaction from them, and he also wants education because it enables him to obtain a suitable social position, and thus benefit is obtained by consuming goods and services. Therefore, it can be said that there is a close relationship between the level of satisfaction and goods and services, which takes the form of a mathematical function. As follows:

$$U = f(x_1, x_2, \dots, x_N, z)$$

Where:

U: represents the total benefit

$x_1, x_2, \dots, x_N$: The various goods and services required by the consumer

z: Other factors affecting utility, such as income, taste, etc

Economists assume other factors to be constant, so the utility function is written as follows:

$$U = f(x_1, x_2, \dots, x_N)$$

The function can also be written if the focus is on measuring the total benefit that the consumer obtains from consuming one commodity as follows: $U = f(x_1)$

Measuring total utility is considered a personal assessment of the consumer himself, as he is the one who estimates the level of satisfaction he obtained from consuming a certain quantity of a commodity, and therefore this estimate varies from one person to another, from one period of time to another, and from one social environment to another.

1.3.6 Basic assumptions of utility theory

1) The theory of utility assumes the economic maturity of the consumer, meaning that the consumer is rational in his actions and economic behavior, and this assumption results in the consumer achieving maximum satisfaction within the limits of his income and the prevailing prices of goods in the market;

2) Assuming the ability of goods to be divided and divisible, as well as the homogeneity of goods in quality, specifications, and shape, and this is necessary when calculating the resulting utility;

3) The ability to consume goods to be saturated.

2.3.6 Numerical utility theory

In the late nineteenth century, classical economists were interested in measuring utility, so the method of numerical measurement of utility is called traditional utility theory, or numerical utility theory, and it is also called marginal utility theory, because of the importance of the concept of marginal analysis of utility in measuring total utility.

1.2.3.6 Total utility (TU): It is the total amount of utility units that the consumer obtains as a result of consuming different units of a specific good in a specific period of time.

Note : Total utility can be obtained from summing the units of marginal utility.

2.2.3.6 Marginal utility (MU): It is the addition to the total benefit resulting from the consumption of an additional unit of the good.

Marginal utility is also defined as the change in total benefit resulting from a change in the quantity consumed of a specific good in one unit, in a specific unit of time, and it can be calculated as follows:

$$MU_x = \frac{\Delta TU}{\Delta X_i} = \frac{chang \quad in\, total\, utility}{change\, in\, units\, of\, good}$$

If the total utility is a function of the various quantities of the commodity consumed $TU = f(Q_x)$, and this function is assumed to be continuous and derivable, then the marginal utility is defined as the limit of the percentage change in the total benefit and consumption when consumption becomes zero.

Marginal utility is the derivative of total utility

$$MU = \lim_{Q_x \to 0} \frac{\Delta TU}{\Delta Q_x} = \frac{\partial TU}{\partial Q_x}$$

Note: The marginal utility before the saturation limit is positive, and its derivative is negative because the increase in total utility is at a decreasing rate.

Example: The following table shows the total benefit resulting from consuming different units of the good x.

	0	1	2	3	4	5	6	7	8
TU	0	20	38	52	63	72	77	77	70

Graphically represent total utility and marginal utility, and explain their behavior economically.

The solution

1) Calculating the marginal utility of the good:

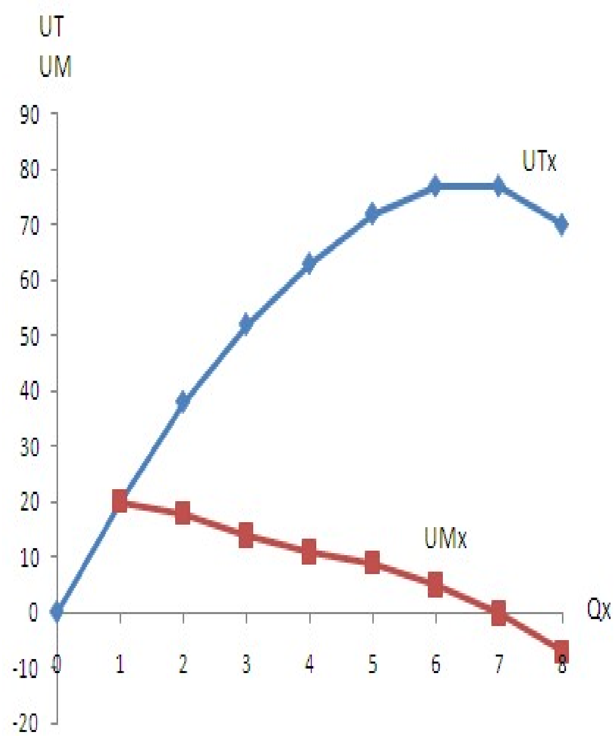
Marginal utility can be calculated using the relationship given previously. The following is an illustrative example of how to calculate it:

$$MU_x = \frac{\Delta TU}{\Delta Q_x} = \frac{TU_2 - TU_1}{Q_2 - Q_1} = \frac{20-0}{1-0} = 20$$

In the same way, the rest of the marginal utility values shown in the following table are calculated:

	0	1	2	3	4	5	6	7	8
TU	0	20	38	52	63	72	77	77	70
MU_x	/	20	18	14	11	9	5	0	-7

Figure (6-1): Graphical representation of both total utility and marginal utility



The total utility increases with the increase in the number of units of the commodity consumed, and this stage continues until the sixth unit is consumed in our example, where the total utility reaches its maximum value, which is the saturation point.

The marginal utility at this stage is decreasing but positive, which makes the total utility increase at a decreasing rate.

Total utility remains constant at its maximum value when consuming the seventh unit. At this stage, the marginal utility has reached zero, meaning that consuming the seventh unit has not added anything to the total utility. Therefore, the marginal utility curve intersects the horizontal axis at the seventh unit.

In the last stage, when the consumer continues to consume the eighth unit, the total benefit decreases, as continuing consumption beyond the saturation limit becomes a source of harm to the consumer, and thus the marginal benefit becomes negative.

3.2.3.6 Law of Diminishing Marginal Utility

The Law of Diminishing Marginal Utility states that consuming successive and homogeneous units of a particular good leads to a decrease in marginal utility until it ceases to exist at the point of saturation, and it becomes negative if consumption continues beyond the point of saturation.

The law of diminishing marginal utility emphasizes that total utility increases at a decreasing rate, and the idea of the consumer achieving the maximum possible utility depends on this law.

4.2.3.6 Consumer equilibrium according to the theory of numerical utility

The consumer is in a state of equilibrium if he is able to purchase the maximum possible quantities of various goods and services, which achieve the maximum benefit for him within the limits of his income and the prevailing prices in the market.

Consumer equilibrium is determined according to the marginal utility method by meeting two basic conditions: the benefit condition and the expenditure condition.

Marginal utilities condition: This condition stipulates that the marginal utilities must be equal for each monetary unit spent on different goods, i.e. equality is achieved between the following ratios:

$$\frac{MUx_1}{P_1} = \frac{MUx_2}{P_2} = \cdots = \frac{MUx_n}{P_n}$$

Where:

MUx_1: the marginal utility of the good x_1;

MUx_2: the marginal utility of the good x_2;

MUx_n: the marginal utility of the good x_n;

P_1, P_2, P_n: commodity prices x_1, x_2, x_n respectively.

Dividing the marginal benefit of a good by its price gives us the corresponding benefit for each monetary unit spent on the good. The consumer buys the good with the largest marginal utility per monetary unit spent. If these utilities are equal for all goods, the marginal utilities condition is met.

If there are only two goods, the marginal utilities condition can be expressed as follows:

$$\frac{MUx_1}{P_1} = \frac{MUx_2}{P_2}$$

Expenditure condition: This condition stipulates that all of the consumer's income is spent on various goods, as he does not save any of it.

The condition for spending on goods is written according to the following equation:

$$\mathrm{R} = \mathrm{P_1 x_1} + \mathrm{P_2 x_2} + \cdots + \mathrm{P_n x_n}$$

Where:

R: consumer income;

P_1, P_2, P_n: commodity prices x_1, x_2, x_n respectively.

If there are only two goods, the spending condition is written as follows:

$$\mathrm{R} = \mathrm{P_1 x_1} + \mathrm{P_2 x_2}$$

From this equation, the quantity consumed of one of the two goods can be determined:

$$x_1 = \frac{R - P_2 x_2}{P_1} \quad , \qquad x_2 = \frac{R - P_1 x_1}{P_2} \qquad ,$$

From these two equations it is clear that there is an inverse relationship between the quantities consumed of the two commodities, as changing one of them by an increase leads to a change in the other by a decrease due to the restriction of spending.

Example: Let us have a consumer who consumes two goods x_1 and x_2, at two prices $P_1 = 6$ and $P_2 = 2$ respectively, within the limits of his income, which equals 52 monetary units. The following table shows the marginal utilities resulting from consuming different quantities of both commodities.

X_1	MUX_1	$\frac{MUx_1}{P_1}$	X_2	MUX_2	$\frac{MUx_2}{P_2}$
1	120	20	1	64	32
2	108	18	2	60	30
3	96	16	3	50	25
4	84	14	4	38	19
5	72	12	5	34	17
6	60	10	6	28	14
7	48	8	7	26	13
8	36	6	8	20	10
9	30	5	9	18	9
10	24	4	10	16	8

It is clear from the table that there are three cases in which the marginal utility of the monetary unit is equal. They represent possible conditions for consumer equilibrium, but there is only one case that achieves equilibrium, which is the one that fulfills the spending condition as well.

The following table shows the case that fulfills both conditions.

X_1	X_2	$\frac{MUx_1}{6} = \frac{MUx_2}{2}$	$52 = 6X_1 + 2X_2$	Remarks
4	6	$\frac{84}{6} = \frac{28}{2} = 14$	$36 = 6 \times 4 + 2 \times 6$	Income is greater than spending
6	8	$\frac{60}{6} = \frac{20}{2} = 10$	$52 = 6 \times 6 + 2 \times 8$	Income equals spending
7	10	$\frac{48}{6} = \frac{16}{2} = 8$	$62 = 6 \times 7 + 2 \times 10$	Spending is greater than income

From the table it is clear that the consumer's equilibrium is achieved by consuming 6 units of the commodity X_1 and 8 units of the commodity X_2, where the marginal monetary utilities of the two commodities are equal, and all income is also spent.

As for the first and third cases in the table, they do not achieve equilibrium despite the marginal utility condition being met, because the spending condition, which stipulates that all income must be spent, has not been met. In the first case, a portion of the income remains with the consumer because spending is less than the available income, and in the third case, the level of spending is greater than income, and therefore the consumer cannot purchase this dual quantity of the two goods.

3.3.6 Ordinal utility

The theory of numerical utility has been subjected to criticism, especially with regard to the hypothesis of utility for numerical measurement, as in reality it is difficult to measure utility numerically because it is subject to the personal evaluation of the consumer, and the assumption of homogeneity and divisibility of goods is unrealistic, which makes comparison difficult between the marginal benefits of successive units of the commodity consumed.

Economists presented an alternative based on the assumption that the consumer is able to arrange his preferences in descending order according to their importance to him, and they are analyzed using indifference curves.

1.3.3.6 Basic assumptions in consumer preferences

The hypothesis of perfection: means that the consumer has the ability to differentiate and choose between different groups of goods.

The assumption of infringement: means that the consumer's preferences and choices are characterized by rationality, and non-contradiction. For example, if the consumer prefers commodity group A over group B and prefers group B over group C, he will inevitably prefer group A over group C, otherwise he will be characterized by contradiction and irrationality.

The hypothesis of non-saturation: means that the consumer prefers many goods to few goods, that is, he prefers the most commodity group, which is the one in which the number of units of goods is greater than other commodity groups.

2.3.3.6 Indifference curves

Consumer preference hypotheses enable discussion of the concept and properties of indifference curves as an appropriate analytical tool to describe different consumer preferences and choices according to ordinal utility theory.

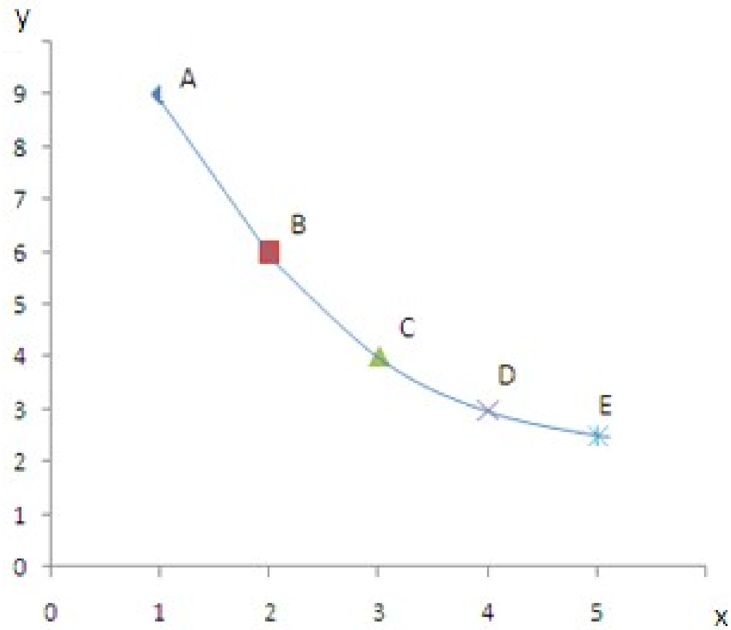
Definition of the indifference curve: It is the graphical representation of the different pairs of goods x and y, which give the consumer the same level of satisfaction (same total utility).

Example: The following table shows five combinations of the two goods x and y, so that each combination gives the consumer the same total utility with the rest of the combinations.

Commodity groups	A	B	C	D	E
X	1	2	3	4	5
Y	9	6	4	3	2.5

By representing the quantities of commodity X on the horizontal axis. and the corresponding quantities of good y on the vertical axis. Then connecting the points with a line, we get an indifference curve, where each point of this curve represents a pair of two commodities , because there is no preference for one point over another for the consumer, and from this the name of the indifference curve is derived.

Figure (6-1): Indifference curve



Properties of indifference curves

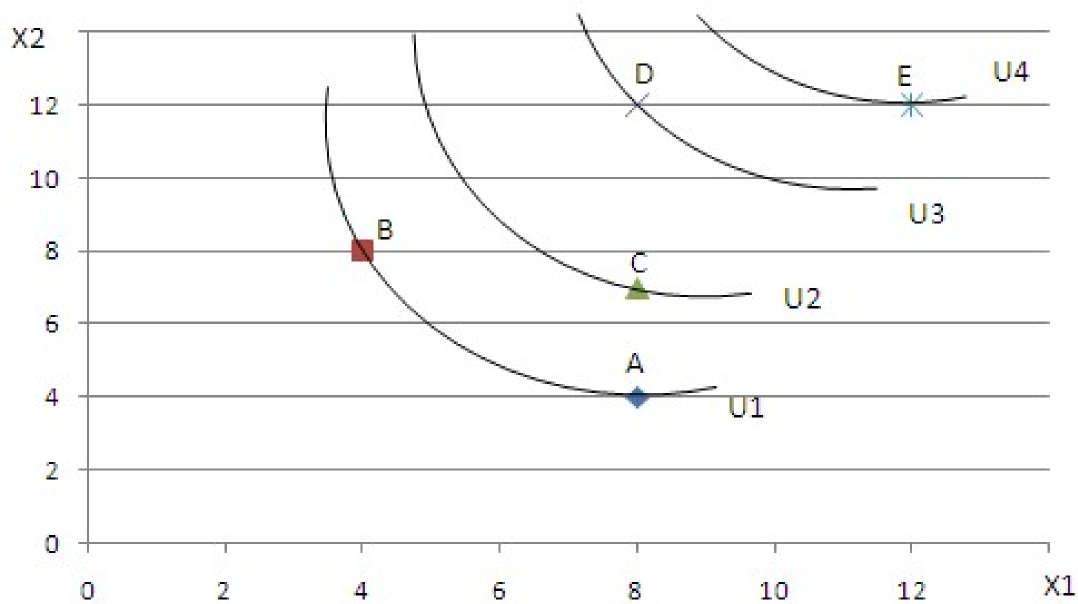
1) The presence of a map of indifference curves: Through the previous hypotheses, it was found that the consumer prefers the group of goods with the most utility over the group with the least utility. The graphical representation that reflects this property is the presence of an infinite number of indifference curves, and the higher the indifference curve moves, the more it indicates an increase in utility for the consumer.

The following table includes commodity binaries that belong to different indifference curves.

Commodity groups	X1	X2
A	8	4
B	4	8
C	8	7
D	8	12
E	12	12

By representing the commodity groups shown in the table and drawing the indifference curves that pass through them, we obtain an indifference map.

Figure (6-2) : Indifference map



We note that the two commodities A and B lie on the same indifference curve U1 because they give the same level of satisfaction. As for the other binaries, they lie on higher indifference curves, because they give a greater level of satisfaction, and the higher the indifference curve is, the greater the total utility it gives. Therefore, the consumer prefers the dual commodity E because it is located on the highest curve U4, and thus it gives him the greatest utility.

2) The slope of the indifference curve is negative

By this we mean that the function $y = f(x)$, which is the function of the indifference curve, accepts the first-order derivative, where:

$$\frac{\partial y}{\partial x} < 0$$

To prove this property, we assume that the function of total utility is $U = f(x, y)$، and when moving from one point to another point on the same indifference curve, then $dU = 0$.

$$dU = MU_x dx + MU_y dy = 0$$

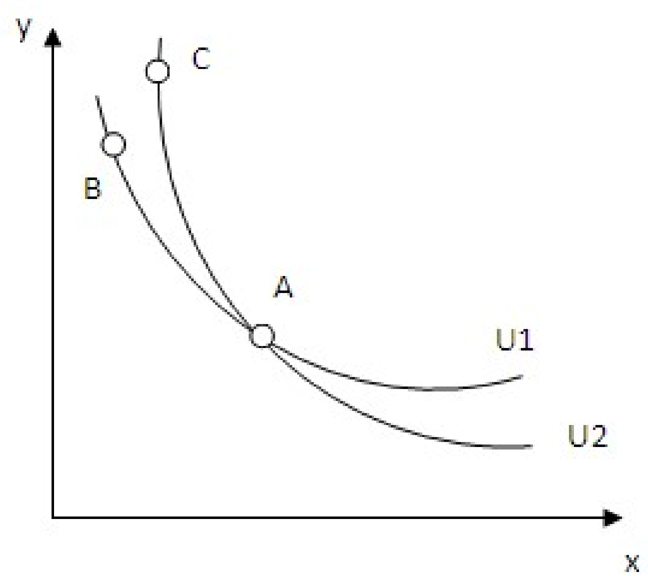
And from that:

$$\frac{dy}{dx} = -\frac{UM_x}{UM_y}$$

Since the slope of the indifference curve $\frac{dy}{dx}$ is negative, the indifference curve slopes from left to right from up to down.

3) Indifference curves do not intersect

This property can be illustrated by the following figure, which assumes the intersection of two indifference curves:



According to the figure, the two commodity groups A and B give the same total utility because they lie on the same indifference curve U1. Groups A and C also give the same total utility because they lie on the same indifference curve U2.

We conclude from this that the two commodity groups B and C give the same level of satisfaction according to the assumption of encroachment, so they must belong to the same indifference curve, which contradicts the previous figure. To resolve this contradiction, we must assert that the indifference curves do not intersect.

4) Indifference curves are concave

To prove this property, we use the marginal rate of substitution between the two goods (MRS) .

Definition : The marginal rate of substitution of good x for good y (MRS $_{x, y}$), is the number of units of good y that the consumer gives up in order to obtain an additional unit of good x , with the level of satisfaction constant.

The number of units that a consumer is willing to give up continually decreases as the number of units of the other good increases.

The marginal rate of substitution of good x for good y can be measured using the following relationships depending on the availability of appropriate data:

$MRS_{x,y} = -\frac{\Delta y}{\Delta x}$: It is the slope of the indifference curve in absolute value in the case of discrete data.

$MRS_{x,y} = -\frac{\partial y}{\partial x}$: It is the slope in absolute value in the case of continuous data.

$MRS_{x,y} = \frac{UMx}{UMy}$: It is the ratio of the marginal utility of the good X to the marginal utility of the good Y.

$MRS_{x,y} = \frac{Px}{Py}$: It is the ratio of the price of the commodity X to the price of the commodity Y in the case of maximum total utility.

Since the slope of the indifference curve is negative, the marginal rate of substitution is decreasing, i.e. its derivative is negative:

$$\frac{dMR\ _{x,y}}{dx} = \frac{-d^2y}{dx^2} < 0$$

Among them: $\frac{d^2y}{dx^2} > 0$ It is the second derivative of the indifference curve equation, and since it is positive, the indifference curve is concave.

Example: Using the data in the following table, calculate the marginal rate of substitution of good x for good y ($MRS_{x,y}$).

Commodity groups	A	B	C	D	E
X	1	2	3	4	5
Y	9	6	4	3	2.5
$MRS_{x,y}$	-	$\left\|\frac{6-9}{2-1}\right\| = 3$	2	1	0.5

From the table it is possible to read the value of the marginal rate of substitution of the commodity X for the commodity Y. For example, for Case B, the consumer gives up 3 units of the commodity Y in order to obtain one unit of the commodity X. Then the marginal rate of substitution continues to decrease for cases C, D, and E. The reason for the decrease is the importance of the commodity to the consumer. In the beginning, the commodity had great importance to the consumer. In order to obtain the second unit of the commodity X, the consumer was willing to give up 3 units of the commodity Y. In the latter cases, the commodity X becomes available to the consumer, which leads to a decrease in its marginal utilities. The consumer is willing to give up smaller and smaller quantities of the commodity Y in order to obtain a unit of the commodity X.

3.3.3.6 Consumer equilibrium according to the theory of ordinal utility

The consumer desires to obtain the maximum quantities in order to achieve the maximum utility. These desires are represented by indifference curves, while the consumer's ability to purchase depends on his available income, which is known graphically as the income line, or the budget constraint line.

Budget line

It is the line that shows the consumer's capabilities to purchase goods, and each point on it represents a combination of goods that can be obtained at a certain income and certain prices.

Assuming that the consumer spends his entire income R to purchase two goods x and y at prices P_x and P_y respectively. The budget constraint equation is written as follows:

$$R = x\,P_x + yP_y$$

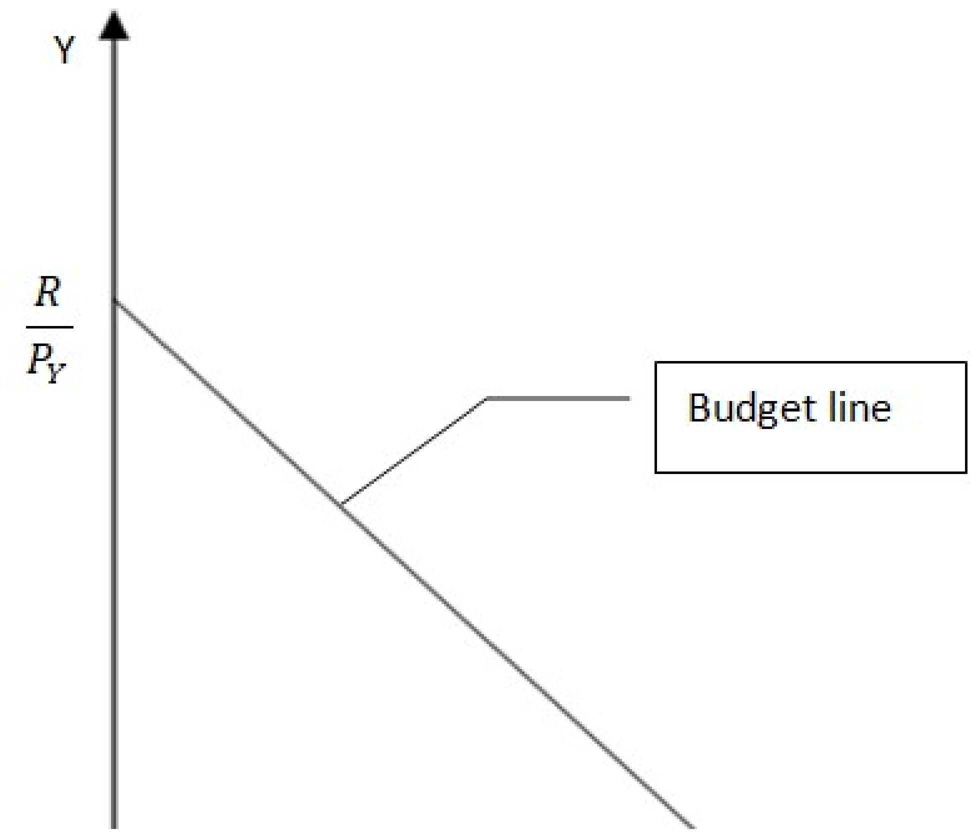
Draw a budget line

The budget line can be drawn by knowing the coordinates of two points on it:

$x = 0 \Longrightarrow y = \frac{R}{P_y}$ Hence the first point $(x, y) = (0, \frac{R}{P_y})$ is the point of intersection with the ranking axis.

$y = 0 \Longrightarrow x = \frac{R}{P_x}$ Hence the second point $(x, y) = (\frac{R}{P_x}, 0)$ is the point of intersection with the axis of the dividers.

Figure (6-3): Budget line



Notes:

- The budget line slopes from left to right, so it has a negative slope. The equation of the budget line can be written as follows:

$$y = R \; -\frac{P_x}{P_y} \; x$$

The slope of the budget line is $\propto = -\frac{P_x}{P_y}$, which is constant along the budget line, unlike the marginal rate of substitution, which changes from one point to another point on the indifference curve.

- The area to the right of the budget line is an area of combinations of goods that the consumer cannot buy because they cost more than his income.
- The area to the left of the budget line is the area of combinations of goods that the consumer can buy and still have a portion of his income left unspent.
- The points of the budget line are combinations of goods that a consumer can purchase so that he spends all of his income.

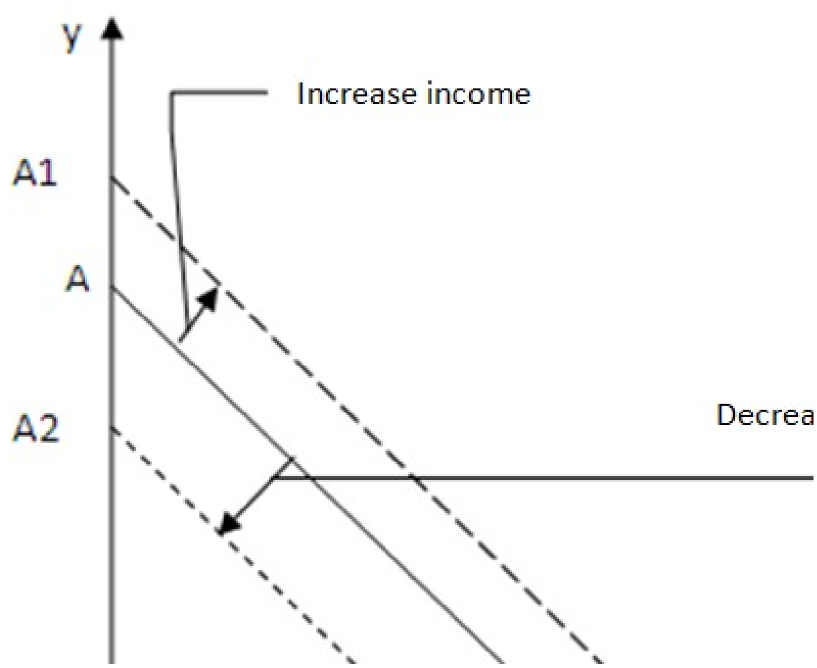
Budget line transmission

The budget line shifts due to a change in income or prices of the two goods.

A) Income changes while prices remain constant

The budget line moves to the right if income increases, and moves to the left if income decreases, but changing income does not affect the slope of the budget line, because the latter relates to the prices of the two goods, which are fixed.

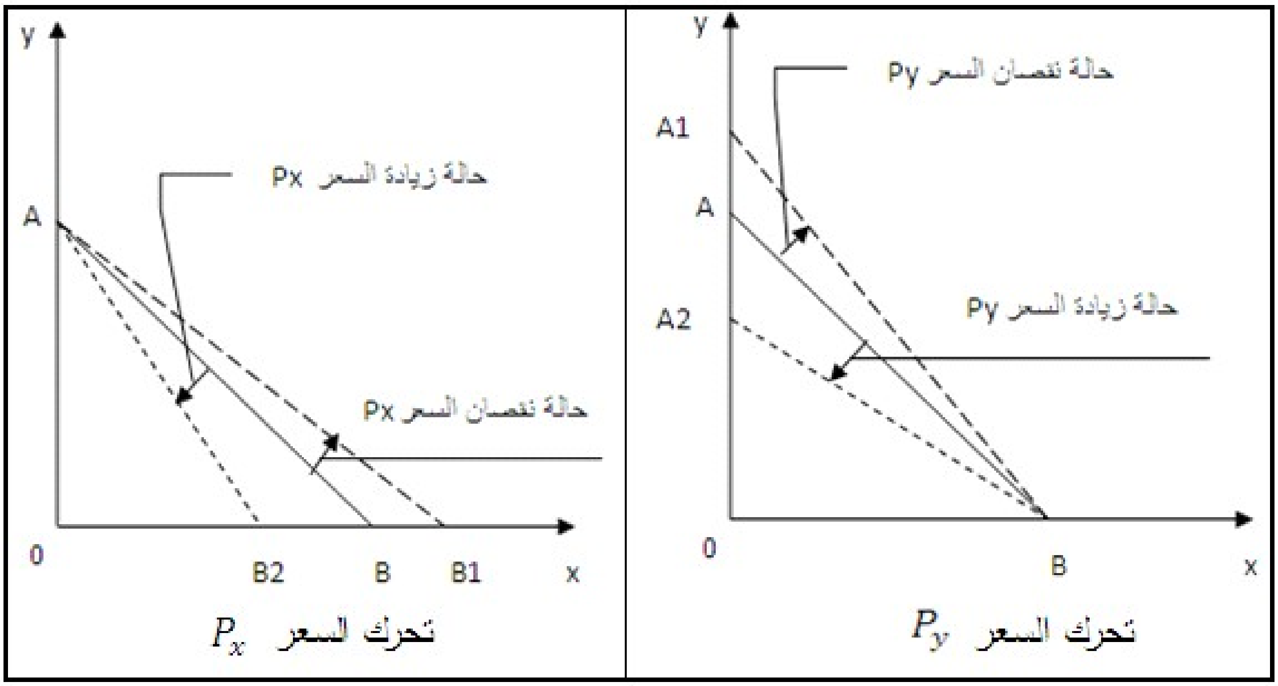
Figure No. (6-4): Budget line transition



B) Change in the price of one of the two goods

The budget line turns if the price of one good changes while the price of the other good and income remain constant. In this case, the slope of the budget line changes whenever one of the two prices changes.

Figure (6-5): Rotation of the budget line



Consumer equilibrium

As we mentioned previously, the rational consumer aims to maximize his utility within the limits of his budget and the prevailing prices in the market. To maximize utility, we use the following methods: the graphical method, the compensation method, and the Lagrange multiplier method.

1. Graphical method: Equilibrium is achieved graphically at the point of contact between the budget line and the indifference curve.

Example: Let the following indifference table be:

X	1	2	3	4	5
y	19	16	14	13	12,5

If you know that $P_x = 10$, $P_y = 5$, and income R = 100 , draw the indifference curve and the budget line, then determine the equilibrium point graphically.

Solution:

- Draw the indifference curve and budget line, and determine the equilibrium point graphically

Based on the data, the equation of the budget line is written as follows:

$$100 = 10x + 5y$$

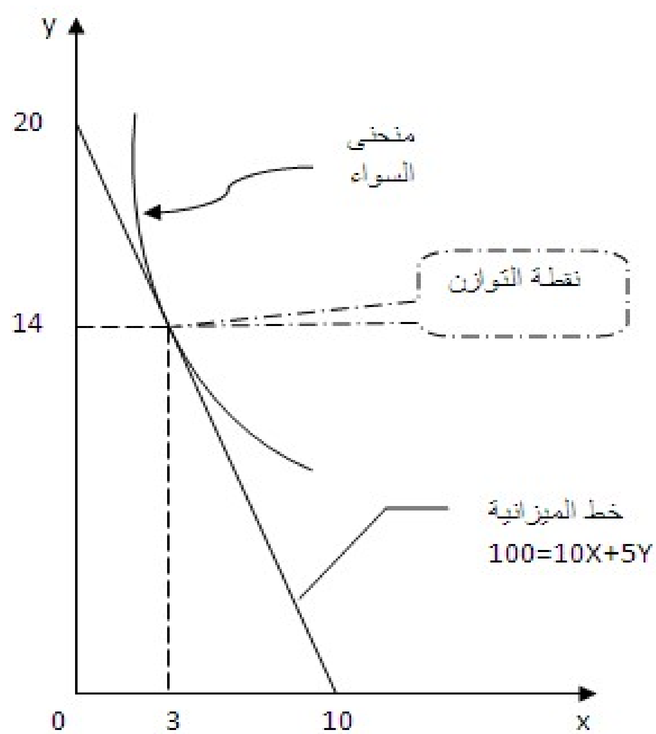
To draw the budget line, it is sufficient to specify two points from it.

When x=0 , y=20, which is the point of intersection of the budget line and the vertical axis.

Since y=0 , x=10 , which is the point of intersection with the horizontal axis.

By drawing the indifference curve from the table given in the example, we get the following figure:

Figure (6-6): Consumer equilibrium graphically



From the graph ,it is clear that the budget line touches the indifference curve when x = 3 and y = 14, where these two quantities of commodities x and y achieve the maximum utility for the consumer while spending all his income, and spending can be verified through the equation of the budget line:

$$100 = 10x + 5y \Rightarrow 100 = 10(3) + 5(14)$$

$$\Rightarrow 100 = 100$$

2 .Compensation method : This method consists of compensating for one of the two quantities of the two goods calculated from the budget equation in the utility function, then searching for the values that maximize the utility function, and we can summarize this in the following stages:

A) We write the quantity of one of the two goods in terms of the quantity of the other good from the budget equation, for example: $y = \frac{R - xP_x}{P_y}$.

b) We substitute into the utility function:

$$U = f(x, y) = f(x, \frac{R - xP_x}{P_y})$$

3) Achieving the conditions of maximizing the utility function (equilibrium conditions):

- The necessary condition for equilibrium is the existence of a stability point, i.e. the first derivative of the utility function is zero $\frac{dU}{dx} = 0$.
- The sufficient condition for equilibrium is that the second derivative of the utility function be negative $\frac{d^2U}{dx^2} < 0$.

Example

Let the utility function of a given consumer be as follows:

$U = x^2y$, and his income is R = 300 , the price of the good X is $P_x = 5$, and the price of the good Y is $P_y = 4$.

Find the equilibrium point of this consumer.

The solution

The budget constraint is written as follows: $R = xP_x + yP_y \Longrightarrow 300 = 5x + 4y$

We write Y in terms of X from the budget constraint: $y = \frac{300-5}{4}$

We substitute Y into the utility function: $U = x^2 \left(\frac{300-5}{4}\right)$

$$= 75x^2 - \frac{5}{4}x^3$$

Necessary condition for balance:

$$\frac{dU}{dx} = 0 \Longrightarrow 150x - \frac{15}{4}x^2 = 0$$

$$\Rightarrow x\left(150 - \frac{15}{4}x\right) = 0$$

$$\Longrightarrow \begin{cases} x = 0 \\ \vee \\ 150 - \frac{15}{4}x = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x = 0 \\ \vee \\ x = 40 \end{cases}$$

If X = 0 then Y = 75, this commodity binary does not fulfill our assumption that the consumer consumes both commodities together .

If X = 40 then Y = 25, this commodity binary achieves consumer equilibrium if the sufficient condition is met .

Sufficient condition for balance:

$$\frac{d^2U}{dx^2} = 150 - \frac{30}{4}x$$

If we substitute X = 0, we find: $\frac{d^2U}{dx^2} = 150 - \frac{30}{4}(0) = 150 > 0$

If X = 0, the sufficient condition for consumer equilibrium is not achieved.

If we substitute X = 40, we find:

$$\frac{d^2U}{dx^2} = 150 - \frac{30}{4}(40) = -150 < 0$$

Hence, commodity binary $(x, y) = (40,25)$ achieves consumer equilibrium.

To ensure that all income is spent, we replace the two equilibrium point quantities in the budget equation:

$$R = 5x + 4y$$
$$= 5 \times 40 + 4 \times 25 = 300$$

Expenditure equals income, from which all income is spent.

3 .Lagrange multiplier method: The Lagrange equation is used to find the consumer‘s equilibrium point from the quantities of the two commodities. It also enables the derivation of the demand functions for the two commodities. It combines the budget line equation with the utility function, and this method is carried out according to the following stages:

a) Formation of the Lagrange function:

$$L = U(x, y) + \lambda(R - x\,P_x - yP_y)$$

Where: λ is the Lagrange multiplier

When spending is equal to income, that is $R - x\,P_x - yP_y = 0$, maximizing the dependent Y is the same as maximizing the utility function $U(x, y)$.

b) Necessary condition for equilibrium:

This condition consists in searching for stability points, and this is achieved through the absence of partial derivatives of the function L for X, Y and λ.

$$\begin{cases} \dfrac{\partial L}{\partial x} = 0 \Rightarrow \dfrac{\partial U}{\partial x} - \lambda P_x = 0 \ldots\ldots\ldots\ldots\ldots (1) \\ \dfrac{\partial L}{\partial y} = 0 \Rightarrow \dfrac{\partial U}{\partial y} - \lambda P_y = 0 \ldots\ldots\ldots\ldots\ldots (2) \\ \dfrac{\partial L}{\partial \lambda} = 0 \Rightarrow R - x\,P_x - yP_y = 0 \ldots\ldots (3) \end{cases}$$

$$(1) \Rightarrow \lambda = \frac{\partial U/\partial x}{P_x}$$

$$(2) \Longrightarrow \lambda = \frac{{\partial U}/{\partial y}}{P_y}$$

From the last two equations we get the equilibrium conditions:

$$\lambda = \lambda \Longrightarrow \frac{{\partial U}/{\partial x}}{P_x} = \frac{{\partial U}/{\partial y}}{P_y}$$

$$\Longrightarrow \frac{UM_x}{P_x} = \frac{UM_y}{P_y} = \lambda$$

This condition shows that the ratio of the marginal utility of each good to its price must equal the common ratio λ, where λ is the marginal utility of the last monetary unit spent.

Sufficient condition

The role of this condition is to prove the existence of a maximum value for the utility function, and it is done by calculating the determinant of the Hessienne matrix, which is the matrix of the second partial derivatives of the function L, symbolized by the symbol H.

Where: the sufficient condition is satisfied if the determinant of the Hessian matrix $|D^2|$ Positive, i.e. $|D^2|>0$.

$$|D^2| = \begin{vmatrix} \frac{\partial^2 L}{\partial xx} & \frac{\partial^2 L}{\partial xy} & \frac{\partial^2 L}{\partial x\lambda} \\ \frac{\partial^2 L}{\partial yx} & \frac{\partial^2 L}{\partial yy} & \frac{\partial^2 L}{\partial y\lambda} \\ \frac{\partial^2 L}{\partial \lambda x} & \frac{\partial^2 L}{\partial \lambda y} & \frac{\partial^2 L}{\partial \lambda\lambda} \end{vmatrix}$$

Derivation of demand functions

and can be derived , X and Y we show this through the following example.

Example: We assume that a certain consumer wants to maximize his utility function: $U = x^{\frac{1}{2}}y^{\frac{1}{2}}$ Under budget constraint $R = x\,P_x + yP_y$.

We first form the Lagrange equation:

$$L = U(x, y) + \lambda(R - x\,P_x - yP_y)$$
$$L = x^{\frac{1}{2}}y^{\frac{1}{2}} + \lambda(R - x\,P_x - yP_y)$$

Find equilibrium conditions:

Dividing (1) by (2) we find:

$$\frac{x^{-\frac{1}{2}}y^{\frac{1}{2}}}{x^{\frac{1}{2}}y^{-\frac{1}{2}}} = \frac{P_x}{P_y} \Longrightarrow x\,P_x = yP_y$$

In compensation for yP_y Equivalent to it in the equation (3):

From it we get the demand function for the commodity X:

$$x = \frac{R}{2P_x}$$

Likewise when compensating $x\,P_x$ for Equivalent to it in the equation (3), we get the demand function for the good Y:

$$y = \frac{R}{2P_y}$$

We note that the demand functions for the two commodities show the existence of a direct relationship between quantities and income, and the existence of an inverse relationship between quantities and prices, which is consistent with the law of demand.

Example:

Let a consumer's utility function be: $U = \frac{1}{2}xy^2$

a. Determine the demand functions for the two goods Y and Y.

b. If $P_x = 1$ ، $P_y = 3$ *and* $R = 16$, determine the consumer's equilibrium point.

The solution

a. demand functions:

We first form the Lagrange equation: $L = \frac{1}{2}xy^2 + \lambda(R - x\,P_x - yP_y)$

Then we look for the equilibrium conditions:

- **Necessary condition**

$$\begin{cases} \frac{\partial L}{\partial x} = 0 \Rightarrow \frac{1}{2}y^2 - \lambda P_x = 0 \dots\dots\dots\dots\dots (1) \\ \frac{\partial L}{\partial y} = 0 \Rightarrow xy - \lambda P_y = 0 \dots\dots\dots\dots\dots (2) \\ \frac{\partial L}{\partial \lambda} = 0 \Rightarrow R - x\,P_x - yP_y = 0 \dots.\dots (3) \end{cases}$$

Dividing (1) by (2) we find: $\frac{\frac{1}{2}y^2}{xy} = \frac{P_x}{P_y} \Rightarrow 2x\,P_x = yP_y$

In compensation yP_y for Equivalent to it in the equation (3):

From it we get the demand function for the commodity X:

$$x = \frac{R}{3P_x}$$

Likewise when compensating $x\,P_x$ for Equivalent to it in the equation (3) we get the demand function for the good Y:

$$y = \frac{2R}{3P_y}$$

B. Determine the equilibrium point

Substituting the values , $P_x = 1$ ، $P_y = 3$ *and* $R = 16$, into the demand functions we obtain the equilibrium point:

$$x = \frac{R}{3P_x} = \frac{16}{3\times1}$$

$$x^* = 5{,}33$$

$$y = \frac{2R}{3P_y} = \frac{2\times 16}{3\times 3}$$

$$y^* = 3{,}55$$

- **Sufficient condition:**

We define the Hessian matrix as follows:

$$H = \begin{pmatrix} \frac{\partial^2 L}{\partial xx} & \frac{\partial^2 L}{\partial xy} & \frac{\partial^2 L}{\partial x\lambda} \\ \frac{\partial^2 L}{\partial yx} & \frac{\partial^2 L}{\partial yy} & \frac{\partial^2 L}{\partial y\lambda} \\ \frac{\partial^2 L}{\partial \lambda x} & \frac{\partial^2 L}{\partial \lambda y} & \frac{\partial^2 L}{\partial \lambda\lambda} \end{pmatrix} = \begin{pmatrix} 0 & y & -P_x \\ y & x & -P_y \\ -P_x & -P_y & 0 \end{pmatrix}$$

Then we calculate the determinant of the hessian matrix:

$$|\mathrm{D}^2| = \begin{vmatrix} 0 & \mathrm{y} & -\mathrm{P_x} \\ \mathrm{y} & \mathrm{x} & -\mathrm{P_y} \\ -\mathrm{P_x} & -\mathrm{P_y} & 0 \end{vmatrix} = 0\begin{vmatrix} \mathrm{x} & -\mathrm{P_y} \\ -\mathrm{P_y} & 0 \end{vmatrix} - \mathrm{y}\begin{vmatrix} \mathrm{y} & -\mathrm{P_y} \\ -\mathrm{P_x} & 0 \end{vmatrix} - \mathrm{P_x}\begin{vmatrix} \mathrm{y} & \mathrm{x} \\ -\mathrm{P_x} & -\mathrm{P_y} \end{vmatrix}$$

$$= -\mathrm{y}\left[(\mathrm{y}\,.\,0) - (-\mathrm{P_x}\,.\,-\mathrm{P_y})\right] - \mathrm{P_x}\left[\left(\mathrm{y}\,.\,-\mathrm{P_y}\right) - (-\mathrm{P_x}\,.\,\mathrm{x})\right]$$

$$= \mathrm{P_x y P_y} + \mathrm{P_x y P_y} - \mathrm{x P_x^2}$$

$$= \mathrm{P_x}\left(2\mathrm{y P_y} - \mathrm{x P_x}\right) = 1.(2.(3{,}55).3 - (5{,}33).1)$$

$$\Rightarrow |\mathrm{D}^2| = 15{,}97$$

We note that the determinant of the Hessian matrix is positive $|D^2| = 15{,}97 > 0$ and therefore the sufficient condition for equilibrium is met.

Determine maximum utility

$$U = \frac{1}{2}xy^2 = \frac{1}{2}(5{,}33)(3{,}55)^2$$

$$\mathrm{U}^* = 33{,}6$$

4.3.3.6 Substitution effect and income effect (Slatsky analysis)

A change in the price of a commodity causes a change in the consumer's real income, and then a change in the consumer's purchasing power, resulting in a change in the volume of consumption of commodities as well as the volume of utility.

Suppose the price of good x decreases. Which leads to an increase in the quantity demanded, but a decrease in the price of commodity x, also causes an increase in the consumer's purchasing power, that is, an increase in real income, which motivates the consumer to increase the required quantities of goods x and y, thus increasing his total utility. We will show below how to find the substitution effect and the income effect mathematically, and for this we assume that:

R represents income, P_x^0 *and* P_y^0 the original prices of the two goods x and y, respectively, P_x^1 is the new price of x, while the price of the second good remains constant.

According to these data, the demand functions for the two commodities can be expressed as follows:

Original order: $(x, y)^0 = (x, y)(P_x^0, P_y^0, R)$

Final order: $(x, y)^1 = (x, y)(P_x^1, P_y^0, R)$

We need to calculate an intermediate demand that makes purchasing power constant, and for that we assume R^s income that gives the same purchasing power that was before the price of commodity x changed , so it is:

$$R^s = P_x^1 x^0 + P_y^0 y^0$$

So we get the demand corresponding to this income:

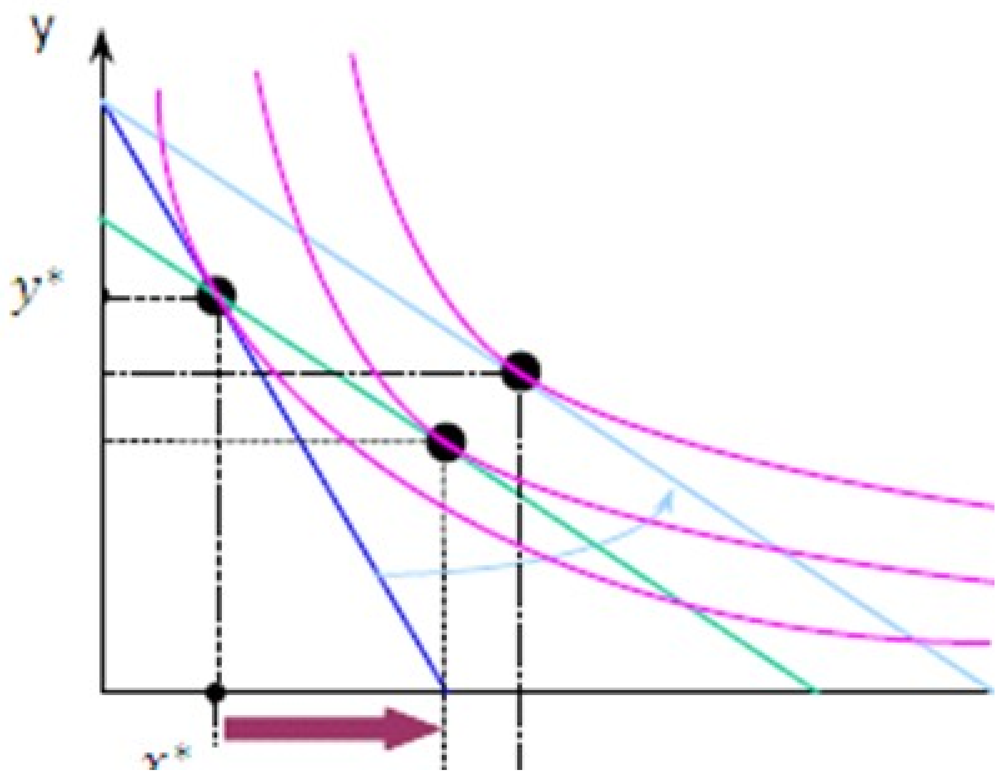
$$(x, y)^s = (x, y)(P_x^1, P_y^0, R^s) = (x, y)^s(P_x^1, P_y^0, x^0, y^0)$$

Finally we get:

- Substitution effect: $SE = (x, y)^s - (x, y)^0$

- Income effect: $IE = (x, y)^1 - (x, y)^s$

Figure 6-7: Substitution effect and income effect according to Slatsky



We assume that the utility function takes the following form:

$$U = x^{\alpha} y^{1-\alpha}$$

So the demand functions for the two commodities are:

$$x = \alpha \frac{R}{P_x}$$

$$y = (1-\alpha) \frac{R}{P_y}$$

With P_x^0 *and* P_x^1 are the original and new prices of commodity x , respectively.

$$y^0 = y^1 = y = (1-\alpha)\frac{R}{P_y} \qquad x^1 = \alpha\frac{R}{P_x^1} \qquad x^0 = \alpha\frac{R}{P_x^0}$$

$$R^s = P_x^1 x^0 + P_y y = P_x^1 \alpha\frac{R}{P_x^0} + P_y(1-\alpha)\frac{R}{P_y}$$

$$R^s = \left[\alpha\frac{P_x^1}{P_x^0} + (1-\alpha)\right]R$$

$$x^s = \alpha\frac{R^s}{P_x^1} = \alpha\frac{R}{P_x^1}\left[\alpha\frac{P_x^1}{P_x^0} + (1-\alpha)\right] = \alpha^2\frac{R}{P_x^0} + \alpha(1-\alpha)\frac{R}{P_x^1}$$

$$x^s = \alpha x^0 + (1-\alpha)x^1$$

Finally, we get the substitution effect and the income effect:

- Substitution effect: $SE = x^s - x^0 = \alpha x^0 + (1-\alpha)x^1 - x^0 = (1-\alpha)(x^1 - x^0)$
- Income effect: $IE = x^1 - x^s = x^1 - [\alpha x^0 + (1-\alpha)x^1] = \alpha(x^1 - x^0)$

7 .Product behavior theory

This theory deals with production and its relationship with the elements of production. The owner of an institution that seeks to maximize profit is not only concerned with the demand side and sales revenues, but is also concerned with achieving economic efficiency in production, that is, production at the lowest possible cost. The facility is an organizational entity that works to exploit a group of production elements in the optimal way, which achieves the maximum possible profit. Before delving into the theory of product behavior, we will discuss some important concepts.

1.7 Production efficiency

Efficiency has two meanings: technical efficiency and economic efficiency in production, as the first is achieved when obtaining the maximum possible production using a certain amount of resources. The second means achieving a certain amount of production at the lowest possible cost. In light of competition, when the prices of produced goods are fixed, the establishment seeks to reduce the cost of the unit produced to the lowest possible level in order to maximize profit. Therefore, maximizing profit includes achieving both efficiencies together, and economic efficiency is a necessary condition and evidence of achieving technical efficiency, but the opposite is not necessarily true. .

2.7 Short term and long term

The short run is the period that is not sufficient for the establishment to change all the elements of production, so at least one element of production remains constant while the rest of the elements change. The long run is the period that allows all elements of production to be changed, which is why it is called the planning run.

3.7 The relationship between production and factors of production

We are mainly interested in identifying two important relationships. The first is known as the law of diminishing marginal productivity of variable production factors or the law of diminishing returns. It depicts the relationship between the increase in one of the variable production factors and total production with the rest of the productive factors remaining constant. It is a relationship linked to the level of activity of the establishment in the short term, and it is useful in Choosing the optimal mix of production factors as well as the optimal level of production in this range. The second relationship links production to the factors of production and is known as the return on scale or return to scale. It depicts the relationship between the change in the quantities of the factors of production combined and total production in the long run, with the change of all productive elements. To understand these two relationships, we first begin by learning about the technical relationship between production and the factors of production in the short run.

4.7 Short-run production function

The short-run production function shows the purely technical relationship between production and the factors of production. This relationship can be formulated by taking only two elements for the sake of simplicity: labor L as a variable factor and capital K as a fixed factor: $Q = f(K^0, L) = g(L)$

Where K^0 is the fixed quantity of factor K.

This mathematical formula is read as saying that the quantity of production is a function that depends on the input quantities of the labor component used in the production of this commodity. The rest of the production factors did not appear because they are fixed and do not affect production quantities.

1.4.7 Law of diminishing returns

The law of diminishing returns governs the production process in the short run, and stipulates that the marginal production of any productive element must decrease with the increase in the quantities used of this productive element, given the stability of the other factors of production.

2.4.7 Total Production (TP) : It is the total of the commodity produced by the establishment when using different levels of the variable production element while the quantities used of the other production elements remain constant. That is, it is the maximum amount that can be produced of the commodity after mixing the variable production element with the fixed production elements.

$$TP = Q = f(K, L, \dots)$$

3.4.7 Average production of labor (AP_L) : It means the average amount that each additional unit of the variable production element (labor) adds to the total production, and it can be obtained by dividing the total production by the number of workers employed.

$$AP = \frac{TP}{L} = \frac{Q}{L}$$

4.4.7 Marginal production of labor(MP_L) : It is what the last added unit of the variable production element (labor) adds to total production, that is, it is the change in total production resulting from changing the variable production element (labor) by one unit. In other words, it is the partial derivative of the production function for a specific productive element (labour).

$$MP_L = \frac{\Delta TP}{\Delta L} = \frac{\Delta Q}{\Delta L}$$

or:

$$MP_L = \frac{\partial Q}{\Delta L}$$

Note: Marginal product can be measured geometrically by the slope of the tangent to the total product curve at the point at which marginal product is to be measured.

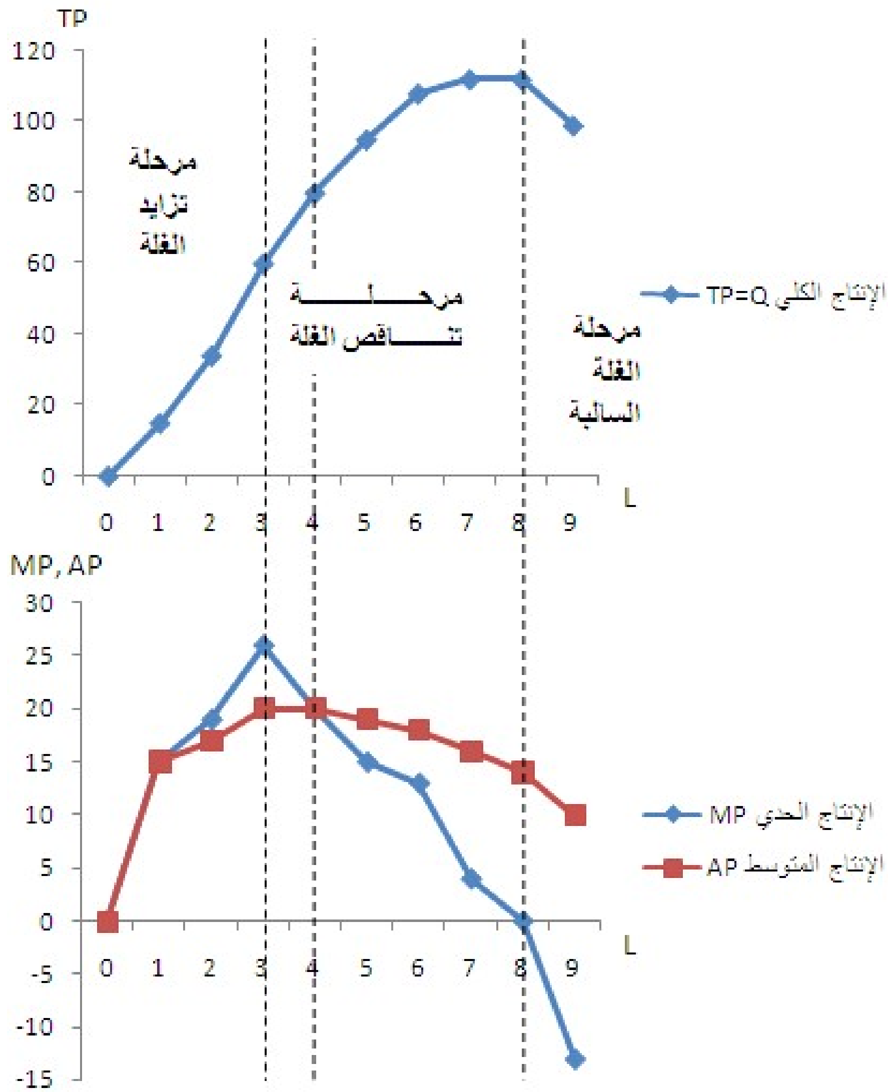
For a deeper understanding of the relationship between total production and other productive elements expressed in the short-run production function, we look at the following example about the production of bags in a factory, using various production elements that are considered fixed, except for the labor element, which is variable, as the table below shows.

Table 7-1: Short-run production function for bags

Number of Workers	Total production TP=Q	Marginal production MP	Average production AP
0	0	0	0
1	15	15	15
2	34	19	17
3	60	26	20
4	80	20	20
5	95	15	19
6	108	13	18
7	112	4	16
8	112	0	14

The data of this table can be translated into a graphical form that shows more clearly the relationship between total production, marginal production, and average production across the different stages of production.

Figure 7-1 : The path of total production, marginal production and average production



It is clear from the table and graph that the production function in the short run goes through three stages.

1) The stage of increasing yields : It starts from the origin and ends at the third unit of labor, during which the total output increases at an increasing rate, where the rate of its increase exceeds the rate of increase in the labor units. The marginal product is also increasing, and reaches its maximum at the end of this stage, and this point is called the inflection point, where the slope of the tangent to the total production curve is the greatest possible. In this stage, average output is increasing, but at a rate less than the rate of increase of marginal production.

2) The stage of diminishing returns: It begins from the third unit of labor to the eighth unit, in which total production is increasing but at a decreasing rate, that is, at a rate less than the rate of increase in labor units, because the marginal product is decreasing from the beginning of this stage until it ceases to exist at its end, and then production is The aggregate is at its maximum, as the slope of the aggregate production curve at this point is zero. Average production at this stage is increasing and reaches its maximum when the tangent to the total product curve passes through the origin, where marginal production equals average production of labour. After that, average production decreases, but it remains positive and greater than marginal production.

3) The stage of negative yields: It starts from the eighth unit of labor to the ninth unit, during which the total production is decreasing, because the marginal production of labor is negative. The average production is decreasing but positive.

5.7 Long-term production machine

The long-run production function expresses the relationship between the volume of production of a particular commodity and the quantities used from the factors of production combined. The behavior of this relationship is governed by the law of returns to scale. The long-run production function can be written assuming only the two components of production, labor and capital, as follows :

$Q = f(K, L, \dots)$, Where Q is the quantity of production.

1.5.7 Law of returns to scale

In the long run, as we mentioned previously, all the elements of production are variable and when all the elements of production increase in certain proportions, that is, when the size of the production process increases, the volume of production changes and this results in a yield (return) called the yield of volume (return on scale), and it takes three cases:

1) The case of increasing returns to scale: This means that the percentage of increase in total production is greater than the percentage of increase in the quantities of production factors used.

2) The state of constant return to scale : This means that the percentage of increase in total production is equal to the percentage of increase in the quantities of production factors used.

3) The case of diminishing returns to scale : If the quantities of factors of production increase by a certain percentage, total production increases by a smaller percentage.

2.5.7 Isometric yield curves

The isoquant is the geometric solution of the sum of possible combinations of the two alternative factors of production, labor and capital, that give the same quantity of a given good produced at any point on this curve.

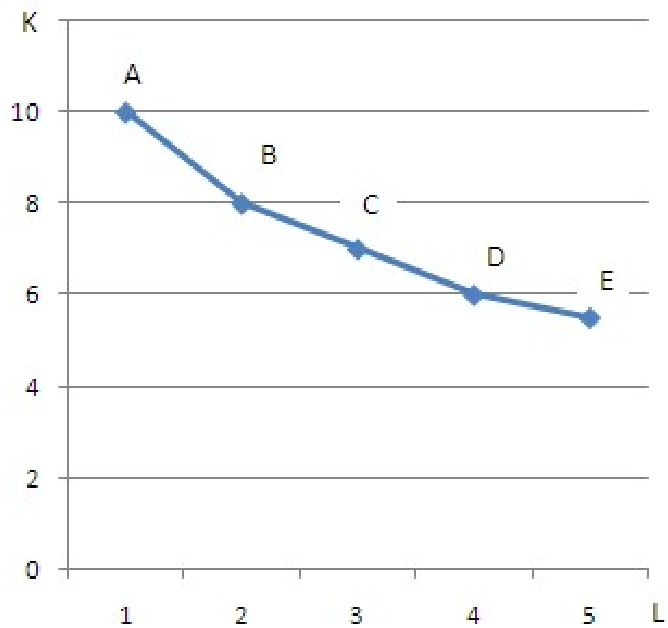
Example

The following table shows the different combinations of the two components of production, labor L and capital K, to produce the same quantity of good x.

combinations	Production volume	Labor	
A	200	1	
B	200	2	
C	200	3	
D	200	4	

By representing the data of this table on a parameter, where the units of labor are represented on the horizontal axis, and the units of capital on the vertical axis, we obtain points, each of which represents a combination of the table, and by connecting these points we obtain the isoquant curve .

Figure No. (7-2): Isometric output curve

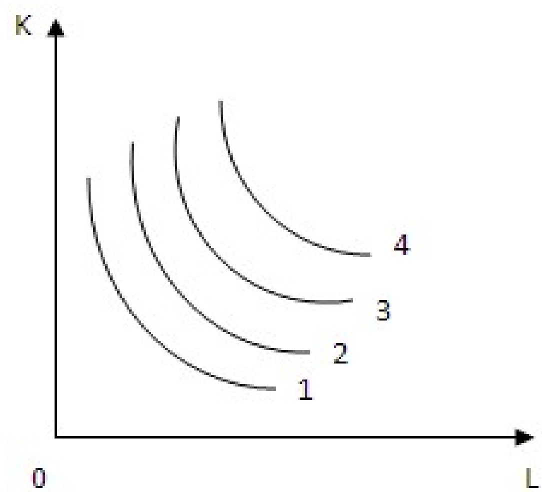


1.2.5.7 Characteristics of isoquants

Isometric curves are similar in mathematical and graphical properties to indifference curves.

1) Isometric output map **:** It is a set of isometric output curves, each of which expresses a different level of production from the other, and the highest curve gives the largest production quantity and is the preferred one for the producer.

Figure No. (7-2): Equal product map



2 (Moving along the same output curve gives the same level of output.

3) The further the isoquant curve is from the origin, the greater the level of production it expresses.

4) Isometric product curves do not intersect.

5) Isometric product curves have a negative slope.

6) Isometric output curves are concave, which means that the rate of technical substitution between the two elements of production is decreasing, and since the curve has a negative slope, it slopes from the left to the right, and it expresses that the quantities used of one element of production decrease as the quantities of the other element of production increase.

2.2.5.7 Marginal rate of technical substitution (MRTS):

It is a rate that measures the necessary decrease in the quantity of a productive element in order to obtain an additional unit of another productive element, while maintaining the same level of production.

It is expressed by the ratio between the change in the units of one of the two elements (capital) to the change in the units of the other element (labor), that is, it is the slope of the isoquant curve in absolute value, and it is also equal to the ratio between the two marginal productivity.

$$MRTS_{L,K} = \left|\frac{\Delta K}{\Delta L}\right| = \frac{MP_L}{MP_K}$$

This relationship represents the amount of capital that must be given up to obtain an additional unit of labor, in order to maintain the same level of production.

Example

technical substitution of labor for capital when moving from point A to point B.

The solution

we've got:

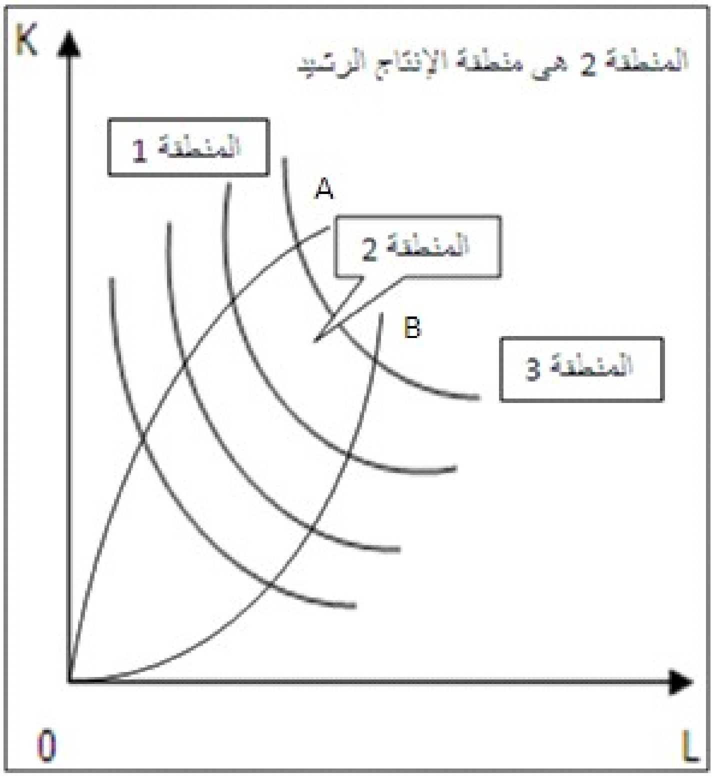
$$MRTS_{L,K} = \left|\frac{\Delta K}{\Delta L}\right|$$

$$MRTS_{L,K} = \left|\frac{8-10}{2-1}\right| = 2$$

To move from point A to point B , the producer must give up two units of capital to replace them with one unit of labor, in order to maintain the same level of production.

3.2.5.7 Rational production area : It is the area in which the optimal combination of production factors is achieved.

Figure No. (7-3): Rational production area



The second region is the rational production region, defined by the productivity edge curves OA and OB , and characterized by positive and decreasing marginal productivity of factors of production.

In the first region, the marginal productivity of capital is negative, while in the third region, the marginal productivity of labor is negative, so the slope of the isometric product curves is positive in the first and third regions, and the marginal productivity of the factor most used is negative.

3.5.7 Cost line

It is the line that reflects all combinations of the two factors of production, labor and capital, which can be obtained at a certain amount of costs and under the prevailing prices of these two factors.

Assuming that the producer uses two production factors, namely labor L and capital K, whose prices are P_L and P_K respectively, and for that the producer spends the cost TC , the cost line equation can be written as follows:

$$TC = KP_K + LP_L$$

Draw a cost line

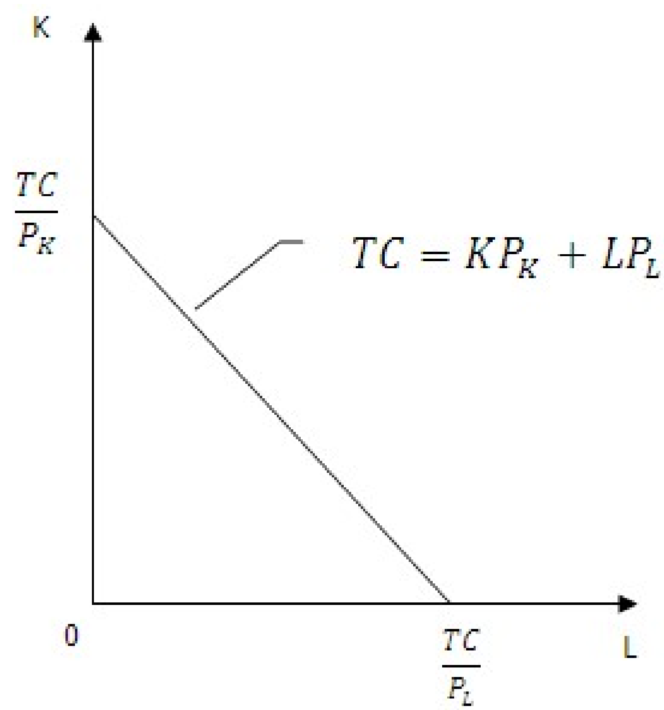
Based on the cost line equation, we determine the points of intersection with the two feature axes

Intersection with the vertical axis

when K = 0 , $TC = LP_L \quad \Rightarrow L = \frac{TC}{P_L}$

when L = 0 , $TC = KP_K \quad \Rightarrow K = \frac{TC}{P_K}$

Figure No. (7-3): Cost line



Slope of cost line:

$$\frac{TC/_{P_K}}{TC/_{P_L}} = \frac{P_L}{P_K}$$

4.5.7 Optimal organization mix

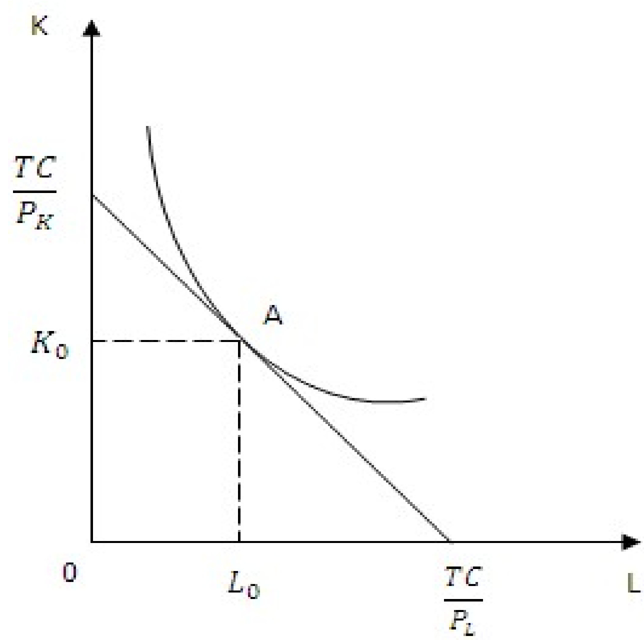
Economic decisions are made by introducing the price factor, which allows the prices of production factors to be compared, so that the organization can reach the optimal combination of these factors.

1.4.5.7 Maximizing outputs

A) Producer equilibrium graphically

Product equilibrium is determined graphically at the point of tangency of the product budget line to the isoquant. At this point, the slope of the isoquant is equal to the slope of the cost line.

Figure 7-4: Producer equilibrium graphically



b) Producer equilibrium mathematically using the Lagrange method

The product equilibrium, which is represented by maximizing production under a certain cost level constraint, is determined using the Lagrange method in stages:

1) Formation of the Lagrange function: It is a function that combines the production function with the cost line function as follows:

$$\mathcal{L} = f(L, K) + \lambda(TC - KP_K - LP_L) = Q + \lambda(TC - KP_K - LP_L)$$

2) The necessary condition for equilibrium: is to obtain a stability point, and this is achieved by the absence of partial derivatives of the function L with respect to both L, K and λ.

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial L} = 0 \Rightarrow \frac{\partial Q}{\partial L} - \lambda P_L = 0 \dots\dots\dots\dots\dots (1) \\ \frac{\partial \mathcal{L}}{\partial K} = 0 \Rightarrow \frac{\partial Q}{\partial K} - \lambda P_K = 0 \dots\dots\dots\dots\dots (2) \\ \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow TC - KP_K - LP_L = 0 \dots\dots (3) \end{cases}$$

$$(1) \Rightarrow \lambda = \frac{\partial Q/\partial L}{P_L}$$

$$(2) \Rightarrow \lambda = \frac{\partial Q/\partial K}{P_K}$$

From the last two equations we get the equilibrium conditions:

$$\lambda = \lambda \Rightarrow \frac{\partial Q/\partial L}{P_L} = \frac{\partial Q/\partial K}{P_K}$$

$$\Rightarrow \frac{MP_L}{P_L} = \frac{MP_K}{P_K} = \lambda$$

This condition states that the ratio of the marginal product of each productive factor to its price must equal the common ratio.

3) Sufficient condition

The role of this condition is to prove the existence of a maximum value for the production function, and it is done by calculating the determinant of the Hessienne matrix, which is the matrix of the second partial derivatives of the Lagrange function L, symbolized by the symbol H.

Where the sufficient condition is satisfied if the determinant of the Hessian matrix $|D^2|$ is positive, i.e. $|D^2| > 0$.

$$|D^2| = \begin{vmatrix} \frac{\partial^2 \mathcal{L}}{\partial LL} & \frac{\partial^2 \mathcal{L}}{\partial LK} & \frac{\partial^2 \mathcal{L}}{\partial L\lambda} \\ \frac{\partial^2 \mathcal{L}}{\partial KL} & \frac{\partial^2 \mathcal{L}}{\partial KK} & \frac{\partial^2 \mathcal{L}}{\partial K\lambda} \\ \frac{\partial^2 \mathcal{L}}{\partial \lambda L} & \frac{\partial^2 \mathcal{L}}{\partial \lambda K} & \frac{\partial^2 \mathcal{L}}{\partial \lambda\lambda} \end{vmatrix}$$

Example

We assume that the production function of an enterprise is given as follows:

$$Q = f(K, L) = 3K + 5L + 6KL$$

If $P_K = 5$, $P_L = 3$ the enterprise's budget is TC= 600, find the enterprise's equilibrium point.

The solution

1) Formation of a Lagrange function

$$\mathcal{L} = f(L, K) + \lambda(TC - KP_K - LP_L) = Q + \lambda(TC - KP_K - LP_L)$$
$$= 3K + 5L + 6KL + \lambda(600 - 5K - 3L)$$

2) Necessary condition

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial L} = 0 \Longrightarrow 5 + 6K - 3\lambda = 0 \dots\dots\dots\dots\dots (1) \\ \frac{\partial \mathcal{L}}{\partial K} = 0 \Longrightarrow 3 + 6L - 5\lambda = 0 \dots\dots\dots\dots\dots (2) \\ \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Longrightarrow 600 - 5K - 3L = 0 \dots\dots\dots (3) \end{cases}$$

$$(1) \Longrightarrow \lambda = \frac{5+6K}{3}$$

$$(2) \Longrightarrow \lambda = \frac{3+6L}{5}$$

From the last two equations we get the equilibrium conditions:

$$\lambda = \lambda \Longrightarrow \frac{5+6K}{3} = \frac{3+6L}{5}$$

$$\Longrightarrow L = \frac{8+15K}{9} \dots\dots\dots\dots (4)$$

Substituting (4) into (3) we find:

$$600 - 5K - 3\left(\frac{8+15K}{9}\right) = 0 \qquad \Rightarrow \boldsymbol{K} = \frac{45}{2} = \mathbf{22,5}$$

By substituting the value of K into equation (4), we find the value of L:

$$\boldsymbol{L} = \frac{8+15(22,5)}{9} = \mathbf{38}$$

The equilibrium point is $(L, K) = (38, 22.5)$, if the sufficient condition is satisfied.

3) Sufficient condition

A) Hessian matrix:

$$H = \begin{pmatrix} \frac{\partial^2 \mathcal{L}}{\partial LL} & \frac{\partial^2 \mathcal{L}}{\partial LK} & \frac{\partial^2 \mathcal{L}}{\partial L\lambda} \\ \frac{\partial^2 \mathcal{L}}{\partial KL} & \frac{\partial^2 \mathcal{L}}{\partial KK} & \frac{\partial^2 \mathcal{L}}{\partial K\lambda} \\ \frac{\partial^2 \mathcal{L}}{\partial \lambda L} & \frac{\partial^2 \mathcal{L}}{\partial \lambda K} & \frac{\partial^2 \mathcal{L}}{\partial \lambda\lambda} \end{pmatrix} = \begin{pmatrix} 0 & 6 & -3 \\ 6 & 0 & -5 \\ -3 & -5 & 0 \end{pmatrix}$$

b) Calculate the determinant of the hessian matrix:

$$|D^2| = \begin{vmatrix} \frac{\partial^2 \mathcal{L}}{\partial LL} & \frac{\partial^2 \mathcal{L}}{\partial LK} & \frac{\partial^2 \mathcal{L}}{\partial L\lambda} \\ \frac{\partial^2 \mathcal{L}}{\partial KL} & \frac{\partial^2 \mathcal{L}}{\partial KK} & \frac{\partial^2 \mathcal{L}}{\partial K\lambda} \\ \frac{\partial^2 \mathcal{L}}{\partial \lambda L} & \frac{\partial^2 \mathcal{L}}{\partial \lambda K} & \frac{\partial^2 \mathcal{L}}{\partial \lambda\lambda} \end{vmatrix} = \begin{vmatrix} 0 & 6 & -3 \\ 6 & 0 & -5 \\ -3 & -5 & 0 \end{vmatrix}$$

$$= -6[(6\,.\,0) - (-3\,.\,-5)] - 3[(6\,.\,-5) - (-3\,.0)]$$

Since the determinant of the Hessian matrix is positive, the second condition is met, including the equilibrium point:

$$(L, K) = (38, 22.5)$$

2.4.5.7 Reducing costs

The organization aims to search for the optimal combination of production workers in order to reduce the cost to the minimum under the constraint of a certain production level. This problem can be expressed mathematically using the Lagrange method as follows:

The equilibrium is determined using the Lagrange method in stages

1) Formation of the Lagrange function: It is a function that combines the production function with the cost line function as follows:

$$\mathcal{L} = TC + \lambda\big(Q - f(L,K)\big) = KP_K + LP_L + \lambda\big(Q - f(L,K)\big)$$

2) The necessary condition for equilibrium: is to obtain a stability point, and this is achieved by the absence of partial derivatives of the function L with respect to both L, K and λ.

$$\begin{cases} \dfrac{\partial \mathcal{L}}{\partial L} = 0 \Rightarrow P_L - \lambda \dfrac{\partial Q}{\partial L} = 0 \ldots\ldots\ldots\ldots\ldots (1) \\ \dfrac{\partial \mathcal{L}}{\partial K} = 0 \Rightarrow P_K - \lambda \dfrac{\partial Q}{\partial K} = 0 \ldots\ldots\ldots\ldots\ldots (2) \\ \dfrac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow Q - f(L,K) = 0 \ldots\ldots..\ldots (3) \end{cases}$$

$$(1) \Rightarrow \lambda = \frac{P_L}{{}^{\partial Q}/_{\partial L}}$$

$$(2) \Rightarrow \lambda = \frac{P_K}{{}^{\partial Q}/_{\partial K}}$$

From the last two equations we get the equilibrium conditions:

$$\lambda = \lambda \Rightarrow \frac{P_L}{{}^{\partial Q}/_{\partial L}} = \frac{P_K}{{}^{\partial Q}/_{\partial K}}$$

$$\Rightarrow \frac{MP_L}{P_L} = \frac{MP_K}{P_K} = \lambda$$

This condition states that the ratio of the marginal product of each productive element to its price must equal the common ratio λ.

3) Sufficient condition

The role of this condition is to prove the existence of a minimum value for the cost function, and it is done by calculating the determinant of the Hessienne matrix, where the sufficient condition is fulfilled if the determinant of the Hessian matrix $|D^2|$ Negative, i.e. $|D^2|<0$.

$$|D^2| = \begin{vmatrix} \frac{\partial^2 \mathcal{L}}{\partial LL} & \frac{\partial^2 \mathcal{L}}{\partial LK} & \frac{\partial^2 \mathcal{L}}{\partial L\lambda} \\ \frac{\partial^2 \mathcal{L}}{\partial KL} & \frac{\partial^2 \mathcal{L}}{\partial KK} & \frac{\partial^2 \mathcal{L}}{\partial K\lambda} \\ \frac{\partial^2 \mathcal{L}}{\partial \lambda L} & \frac{\partial^2 \mathcal{L}}{\partial \lambda K} & \frac{\partial^2 \mathcal{L}}{\partial \lambda\lambda} \end{vmatrix}$$

3.4.5.7 Profit maximization

The firm aims to maximize its profit according to a program that it operates by changing either the level of the budget allocated to purchasing factors of production, or the level of production outputs.

A) The Firm's profit: It is the difference between the total revenue obtained from the sale of outputs and the cost of production.

$$\pi = P \times Q - TC$$

Where: P is the unit price produced, Q is the quantity of production, and TC is the total cost.

When replacing the values of Q and TC, the previous equation takes the following form:

$$\pi = P \times f(L, K) - (KP_K + LP_L)$$

B) The necessary condition for maximizing profit: is to obtain a stability point, and this is achieved by the absence of partial derivatives of the function π for L and K.

$$\begin{cases} \dfrac{\partial \pi}{\partial L} = 0 \Rightarrow P\dfrac{\partial Q}{\partial L} - P_L = 0 \dots\dots\dots\dots\dots (1) \\ \dfrac{\partial \pi}{\partial K} = 0 \Rightarrow P\dfrac{\partial Q}{\partial K} - P_K = 0 \dots\dots\dots\dots\dots (2) \end{cases}$$

$$(1) \Rightarrow P_L = P\frac{\partial Q}{\partial L} = P.MP_L \Rightarrow P = \frac{MP_L}{P_L}$$

$$(2) \Rightarrow P_K = P\frac{\partial Q}{\partial K} = P.MP_K \Rightarrow P = \frac{MP_K}{P_K}$$

From the last two relationships, it becomes clear that the enterprise achieves the greatest profit when the ratio of the marginal productivity of the productive factor to its price is equal to the price of the unit produced.

C) Sufficient condition

This condition consists of achieving the following:

$$|D_1| = \left|P\frac{\partial^2 Q}{\partial LL}\right| < 0$$

$$|D_2| = P^2 \begin{vmatrix} \frac{\partial^2 Q}{\partial LL} & \frac{\partial^2 Q}{\partial LK} \\ \frac{\partial^2 Q}{\partial KL} & \frac{\partial^2 Q}{\partial KK} \end{vmatrix} > 0$$

Where: $|D_1|$ and $|D_2|$ are two prime determinants.

8.7 Cobb-Douglas function

1) Mathematical formula

It is one of the most famous production function formulas used in micro and macro economic analysis, and takes the following form:

$$Q = f(K, L) = AL^{\alpha}K^{\beta}$$

Where: $0 < A \,, \quad 0 < \alpha < 1, \quad 0 < \beta < 1$

Q: volume of production, L: labor, K: capital.

A: Constant indicating the technical (technological) level.

α: Elasticity of production with respect to labor, β elasticity of production with respect to capital.

Regarding the Cobb-Douglas production function, the volume of production Q is a function of the quantities used of the two factors of production, labor and capital, as well as the technical level. Therefore, any change in the technical level affects the level of production.

2) Elasticity of production with respect to factors of production

It is the degree of response of the volume of production to the relative change occurring in one of the production factors used.

- Elasticity of production with respect to labor: It is the ratio of the relative change in the quantity of production to the relative change in labor.

$$E_L = \frac{\%\Delta Q}{\%\Delta L} = \frac{\partial Q}{\partial L} \times \frac{L}{Q}$$

We look for elasticity of production with respect to the labor factor

$$E_L = \frac{\partial Q}{\partial L} \times \frac{L}{Q} = A\alpha L^{\alpha-1}K^{\beta}\frac{L}{Q} = \alpha\frac{Q}{Q}$$

$$E_L = \alpha$$

The constant α in the Cobb-Douglas production function expresses the elasticity of production with respect to the labor, which measures the percentage change in the volume of production resulting from a change in the quantity of labor by 1% .

- Elasticity of production with respect to capital: It is the ratio of the relative change in the quantity of production to the relative change in capital.

$$E_K = \frac{\%\Delta Q}{\%\Delta K} = \frac{\partial Q}{\partial K} \times \frac{K}{Q}$$

We look for the elasticity of production with respect to the capital factor

$$E_K = \frac{\partial Q}{\partial K} \times \frac{K}{Q} = AL^{\alpha}\beta K^{\beta-1}\frac{K}{Q} = \beta\frac{Q}{Q}$$

$$E_K = \beta$$

The constant β in the Cobb-Douglas production function expresses the elasticity of production with respect to the capital component, which measures the percentage change in the volume of production resulting from a change in the quantity of capital by 1%.

3) Law of diminishing returns

It can be proven that the law of diminishing returns applies to the Cobb-Douglas function, and for this we show that the second derivative of the function with respect to the labor is negative.

We've got:

$$Q = f(K, L) = AL^{\alpha}K^{\beta}$$

$$\frac{\partial Q}{\partial L} = A\alpha L^{\alpha-1}K^{\beta}$$

$$\frac{\partial^2 Q}{\partial L^2} = A\alpha(\alpha - 1)L^{\alpha-2}K^{\beta} = A\alpha(\alpha - 1)L^{\alpha}L^{-2}K^{\beta}$$

$$= \alpha(\alpha - 1)\frac{Q}{L^2} < 0$$

Since $0 < \alpha < 1$, then $(\alpha - 1) < 0$ hence the second derivative of the Cobb-Douglas function with respect to labor is negative.

Note: In the same way, it can be proven that the second derivative of the Cobb-Douglas function with respect to capital is negative.

4) Yield of scale

Degree of homogeneity of the Cobb-Douglas function:

We multiply the two factors of production by a constant λ

$$Q = AL^{\alpha}K^{\beta} \Longrightarrow Q_1 = A(\lambda L)^{\alpha}(\lambda K)^{\beta} = A\lambda^{\alpha}L^{\alpha}\lambda^{\beta}K^{\beta}$$
$$= \lambda^{\alpha+\beta}Q$$

Among them is the homogeneous Cobb-Douglas function of degree $\alpha + \beta$, where if we multiply the two production factors by a constant λ, the volume of production increases by a value $\lambda^{\alpha+\beta}$.

Volume Yield Cases:

- If $\alpha + \beta = 1$: the Cobb-Douglas function is homogeneous of the first order, then increasing the two production factors together by a certain percentage results in increasing production by the same percentage, and we express this as a state of constant yield to scale.

- If $\alpha + \beta > 1$: the Cobb-Douglas function is homogeneous to a greater degree than one, then increasing the two production factors together by a certain percentage results in increasing production by a greater percentage, and we express this as a condition of increasing return to scale.
- If $\alpha + \beta < 1$: the Cobb-Douglas function is homogeneous to a degree less than one, then increasing the two production factors together by a certain percentage results in an increase in production by a smaller percentage, and we express this as a state of diminishing returns to scale.

Note: It is usually assumed that $\alpha + \beta = 1$.

4) Optimal production level

We will explain how to find the optimal combination of factors of production according to the Cobb-Douglas function, by using the Lagrange method in the following example.

Example

Let the production function be of the following Cobb-Douglas type:

$$Q = 50L^{\frac{1}{3}}K^{\frac{2}{3}}$$

If $P_L = 4$ ، $P_K = 6$, and total costs $TC = 72$, calculate the largest output that can be achieved.

The solution

Using the Lagrange method

1) Formation of a Lagrange function

$$\mathcal{L} = f(L, K) + \lambda(TC - KP_K - LP_L) = Q + \lambda(TC - KP_K - LP_L)$$
$$= 50L^{\frac{1}{3}}K^{\frac{2}{3}} + \lambda(72 - 6K - 4L)$$

2) Partial derivatives

$$\begin{cases} \dfrac{\partial \mathcal{L}}{\partial L} = 0 \Longrightarrow \frac{50}{3}\frac{K^{\frac{2}{3}}}{L^{\frac{2}{3}}} - 4\lambda = 0 \dots\dots\dots\dots\dots (1) \\ \dfrac{\partial \mathcal{L}}{\partial K} = 0 \Longrightarrow \frac{100}{3}\frac{L^{\frac{1}{3}}}{K^{\frac{1}{3}}} - 6\lambda = 0 \dots\dots\dots\dots\dots (2) \\ \dfrac{\partial \mathcal{L}}{\partial \lambda} = 0 \Longrightarrow 72 - 6K - 4L = 0 \dots\dots\dots\dots (3) \end{cases}$$

$$(1) \Longrightarrow \lambda = \frac{25K^{\frac{2}{3}}}{6L^{\frac{2}{3}}}$$

$$(2) \Longrightarrow \lambda = \frac{50L^{\frac{1}{3}}}{9K^{\frac{1}{3}}}$$

From the last two equations we get the equilibrium conditions:

$$\lambda = \lambda \Longrightarrow \frac{25K^{\frac{2}{3}}}{6L^{\frac{2}{3}}} = \frac{50L^{\frac{1}{3}}}{9K^{\frac{1}{3}}}$$

$$\Longrightarrow 4L = 3K \dots\dots\dots\dots\dots (4)$$

Substituting (4) into (3) we find:

$$72 - 6K - 3K = 0 \qquad \Longrightarrow \boldsymbol{K} = \frac{72}{9} = \mathbf{8}$$

By substituting the value of K into equation (4), we find the value of L:

$$\boldsymbol{L} = \frac{3\times 8}{4} = \mathbf{6}$$

The equilibrium point is $(L,\ K) = (6,\ 8)$.

8 .Production costs

Production costs are defined as the amount of money that an organization bears in order to obtain the production services necessary to achieve the production of a specific good or service during a specific period of time, and the nature of production costs differs in the short period from that in the long period.

Explicit (direct) costs

These are the costs incurred by the establishment in exchange for obtaining the various production elements necessary to produce a certain quantity of goods and services, such as: wages, the price of raw materials, maintenance expenses, electricity,..., and they are recorded in the firm's accounting books.

Implicit costs (indirect)

They are the opportunity costs of the production elements owned by the facility, such as lands and buildings, as well as the time and effort of the organizer, in addition to the opportunity cost of the facility owner's capital that he invested in it, all of which do not pay explicit and direct expenses in order to obtain them. The opportunity costs are estimated at the amount that could have been These elements are obtained from being employed by others.

As a result of this difference in estimating costs according to the accounting concept and the economic concept, it is possible to distinguish between two types of profits:

Accounting profit = revenues - explicit costs

Economic Profit = Revenue – (Explicit Costs + Implicit Costs)

Where: Explicit costs + Implicit costs = Total costs

From the relationship of economic profit, we distinguish three cases

- If revenues equal total costs, there is neither profit nor economic loss
- If revenues are greater than total costs, there is an economic profit
- If revenues are smaller than total costs there is an economic loss

1.8 Costs in the short run

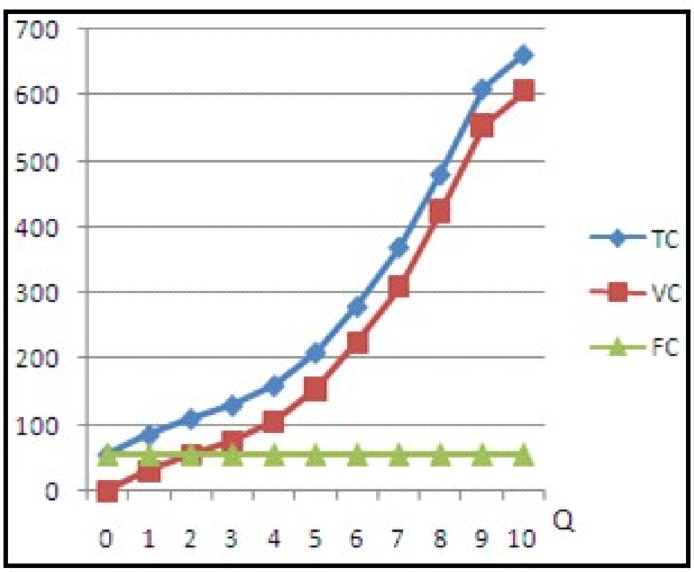
Production costs in the short run consist of fixed costs and variable costs

1.1.8 Fixed costs (FC): These are the costs that are related to the fixed production factors that the organization uses in the production process, and they do not change with the volume of production, and are borne by the organization whether it produces or not, such as building rent, insurance premiums...

2.1.8 Variable costs (VC): These are the costs of the variable production elements necessary to produce a particular good. The facility bears them only if it produces production, but if it does not produce it, it does not bear them.

3.1.8 Total costs (TC): It is the sum of fixed costs and variable costs at each production volume. TC = FC + VC

To study the shape of the fixed, variable and total cost curves, we take an example, where we draw the curves of the various types of costs expressed by the data of the following table in the corresponding figure:



TC	VC	FC	Q
55	0	55	0
85	30	55	1
110	55	55	2
130	75	55	3
160	105	55	4
210	155	55	5
280	225	55	6
370	310	55	7
480	425	55	8
610	555	55	9
662	607	55	10

From the graphic curve it is clear that:

- The curve of fixed costs is parallel to the horizontal axis because they are independent of the volume of production and are constant at all levels of production.

- The variable costs curve starts from the origin because variable costs are related to the volume of production.
- The total costs curve takes the same behavior as the variable costs curve, as it rises above it by a fixed amount that represents the amount of fixed costs at all levels of production.

4.1.8 Costs at the unit level

Firms are interested in determining the cost of one unit of their production because sales are determined by the price of one unit.

1.4.1.8 Average costs

It is the share of the total costs per unit produced and is divided into:

A) Average fixed cost (AFC) : It is the share of fixed costs per unit produced and takes the following formula: $AFC = \frac{FC}{Q}$

b) Average Variable Cost (AVC) : It is the share of variable costs per unit produced and takes the following formula: $AVC = \frac{VC}{Q}$

C) Average Total Cost (ATC) : It is the share of the total costs per unit produced and takes the following formula: $AVC = \frac{FC+VC}{Q} = AFC + AVC$

2.4.1.8 Marginal costs (MC)

It is the amount of change in total costs (or variable costs) as a result of changing the volume of production by one unit. In other words, it is the cost of the last unit produced.

$$MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta(FC+V\ \ )}{\Delta Q} = \frac{\Delta FC}{\Delta Q} + \frac{\Delta VC}{\Delta Q}$$

Since the change in fixed cost is zero $\Delta FC = 0$:

$$MC = \frac{\Delta VC}{\Delta Q}$$

Therefore, any change in total costs in the short run comes from variable costs.

2.8 **Costs in the long run**

In the long period, the production institution is able to change all factors of production, and this depends on the nature of the production process and the extent of specialization of the capital equipment. Therefore, all costs are variable, including the costs that were fixed in the short period. Therefore, it is considered F a continuous variable that is included in each of the production functions $Q = f(L, K, F)$

And the cost equation $C = P_1L + P_K + S(F)$

The expansion channel function $Q = g(L, K, F)$

By solving the three equations, we obtain the long-period cost function at the production level:

$$C = h(Q, F) + S(F)$$

In this last function if we consider F fixed We obtain the cost function in the short period, from which the total cost function in the long run expresses the lowest cost of producing each level of production for a size of the enterprise. Likewise, the marginal costs in the long period represent the increase in total costs that occurs due to the transition from one production size to another. Be more productive with optimized alignment before and after the change.

The total cost is defined geometrically in the long run as the geometric solution to the lowest cost point when the size of the enterprise changes.

1.2.8 Assumptions on which the analysis of average costs in the long period is based

- All factors can change their quantities.
- Some factors are indivisible.
- Increases in the quantities of all factors allow greater specialization in the use of particular units.
- Administrative work cannot be multiplied in the same way that other factors can be increased.
- Stability of technical conditions of production and prices of production factors.

- Successive units of production factors have a single degree of production efficiency.

2.2.8 The relationship between average cost, marginal cost and the laws of yield

To investigate this relationship, we first know cost elasticity, and we can define it as the degree of responsiveness of total costs to a relative change in the volume of production.

$$E = \frac{dT_C}{T_C} / \frac{dQ}{Q} = \frac{dT_C}{dQ} \cdot \frac{Q}{T_C}$$

$$E = \frac{dT_C}{dQ} / \frac{T_C}{Q}$$

Where $\frac{dT_C}{dQ}$ represents marginal cost, $\frac{T_C}{Q}$ represents average cost.

That is, cost elasticity = marginal cost ÷ average cost $E = \frac{MC}{AC}$

So we note that cost elasticity is a relationship linking marginal cost and average cost.

- If $E < 1$: the marginal cost is smaller than the average cost, this happens when production is subject to increasing returns, meaning a relative increase in production is achieved at a lower relative cost.
- If $E = 1$: the marginal cost is equal to the average cost, this happens when production is subject to constant yields, meaning a relative increase in production is achieved with the same relative increase in costs.
- If $E > 1$: the marginal cost is greater than the average cost, this occurs when production is subject to diminishing returns, meaning a relative increase in production is achieved at a greater relative cost.

  On the basis of this measure, we can calculate the elasticity of costs at any production volume, and we can determine the stage to which production is subject. We can also calculate the volume of production at any value of elasticity.

Example: If the total production cost function is: $C = Q^3 - 6Q^2 + 15Q + 2$, Q takes its values from 1 to 6.

- Determine the total fixed cost.
- Determine the average fixed cost.
- Determine the average total cost.
- Determine the total variable cost.
- Determine the average variable cost.
- Determine marginal cost.

The solution

1. The total fixed cost is: $FC = 2$

- Average fixed cost She: $AFC = \frac{2}{Q}$
- The average total cost is: $AC = \frac{C}{Q} = Q^2 - 6Q + 15 + \frac{2}{Q}$
- Total variable cost: $VC = C - F = Q^3 - 6Q^2 + 15Q$
- Average variable cost:

$$\begin{aligned} AVC &= AC - AFC \\ &= Q^2 - 6Q + 15 \end{aligned}$$

- Marginal cost: $MC = \frac{dC}{dQ} = 3Q^2 - 12Q + 15$

Q	C	VC	AVC	FC	AC	MC
1	12	10	10	2	12	6
2	16	14	7	1	8	3
3	20	18	6	0,66	6,66	6
4	30	28	7	0,5	7,5	15
5	52	50	10	0,4	10,4	30
6	92	90	15	0,33	15,3	51

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